Answers to exercises

1. Measurement systems

System functions

1.1. See page 3.
1.2. See page 4.
1.3. See page 4.

System specifications

1.4. The factor $\frac{1}{2}$ refers to half the power transfer; as the power is the square of the voltage or current, half the voltage transfer corresponds to $1/\sqrt{2}$ of the power transfer.

1.5. See pages 10 and 11.

1.6. $v_c = \frac{1}{2}(v_1 + v_2) = 10.2$ V $\rightarrow v_{oc} = 50 \cdot 10^{-3} \cdot 10.2 = 0.51$ V

$v_d = v_1 - v_2 = 0.2$ V $\rightarrow v_{oc} = 50 \cdot 0.2 = 10$ V

(obviously, the gain for a common mode voltage v_c is factor CMRR smaller than that for a differential mode voltage v_d; the polarity of v_{oc} is not known).

$v_o = v_{oc} \pm v_{od} = 10 \pm 0.51$ V. Hence, $9.49 \leq v_o < 10.51$ V.

1.7. $(\mathrm{d}V_o/\mathrm{d}t)_{\max}$ may not exceed the slew rate.

(a)$(\mathrm{d}V_o/\mathrm{d}t)_{\max} = \omega\hat{V}_o = 2\pi f \cdot 100 \cdot 0.1 = 20\pi f \leq 10$ V/μs $= 10^7$ s $\rightarrow f \leq 10^7/20\pi \approx 160$ kHz.

(b)$(\mathrm{d}V_o/\mathrm{d}t)_{\max} = 2\pi\, 10^6 \cdot \hat{V}_o = 2\pi \cdot 10^6 \cdot 100 \cdot \hat{V}_i \leq 10^7$ V/s $\rightarrow \hat{V}_i \leq 10^7/(2\pi \cdot 10^8) \approx 16$ mV.

1.8. $V_{\mathrm{off}}(\max) = V_{\mathrm{off}}(t = 20°) + (\Delta T)_{\max} \cdot$ (temperature coefficient):

$V_{\mathrm{off}}(\max) = 0.5$ mV $+ 60 \cdot 5\mu$V $= 0.8$ mV.

1.9. $x_o(\text{linear}) = \alpha x_i \rightarrow$ non-linearity $= x_o - x_o(\text{linear}) = \beta x_i^2$.

Relative non-linearity $= \beta x_i/\alpha = 0.02 x_i$. The maximum value is $\pm 0.02 \cdot 10 = \pm 20\%$.

2. Signals

Periodical signals

2.1. (a) $x_{pp} = E;\ \ x_m = |x|_m = \frac{4}{6}E;\ \ x_{rms}^2 = \frac{1}{T}\int_0^T x^2 dt = \frac{1}{6}\int_1^5 E^2 dt = \frac{4}{6}E^2 \to x_{rms} = E\sqrt{\frac{2}{3}}$

(b) $x_{pp} = \frac{3}{2}E;\ \ x_m = \frac{1}{6}\left[3\cdot\frac{E}{2} - 2E\right] = -\frac{1}{12}E;\ \ |x|_m = \frac{1}{6}\left[3\cdot\frac{E}{2} + 2E\right] = \frac{7}{12}E;$

$x_{rms}^2 = \frac{1}{6}\left(\int_1^4\left(\frac{1}{2}E\right)^2 dt + \int_4^6(-E)^2 dt\right) = \frac{1}{6}\left(3\cdot\frac{1}{4}E^2 + 2E^2\right) = \frac{11}{24}E^2 \to x_{rms} = E\sqrt{\frac{11}{24}}.$

(c) $x_{pp} = E;\ x_m = |x|_m = \frac{1}{6}\int_0^T x\,dt = \frac{1}{6}\cdot(\text{area } \Delta) = \frac{1}{6}\cdot 2E = \frac{1}{3}E;$

for $1 \le t \le 3: x = x_1 = -\frac{1}{2}E + \frac{1}{2}Et;$ for $3 \le t \le 5: x = x_2 = -\frac{5}{2}E - \frac{1}{2}Et$;

$x_{rms}^2 = \frac{1}{6}\left(\int_1^3 x_1^2 dt + \int_3^5 x_2^2 dt\right) \to x_{rms} = \frac{1}{3}E\sqrt{2}.$

2.2. The crest factor is x_p/x_{rms}; $x_p = A$;

$x_{rms} = ((1/T)\int_0^\tau A^2 dt)^{1/2} = A\sqrt{\tau/T} \to \text{crest factor} = \sqrt{T/\tau}.$

2.3. $\sqrt{T/\tau} \le 10 \to \tau \ge T/100.$

2.4. Sine wave signal: $x_p/x_{rms} \approx 1.41$; $x_{rms}/|x|_m \approx 1.11$.

(a) $V_i = -1.5$ V $\to V_{ind} = |-1.5|\cdot 1.11 = 1.665$ V.

(b) The input is a sine wave, so the indication is $V_{ind} = V_{rms} = \tfrac{1}{2}\sqrt{2}\hat{V}_i = 1.06\,\text{V}$.

(c) $|V_i|_m = 1.5$ V $\to V_{ind} = 1.11\cdot 1.5 = 1.665$ V.

(d) $|V_i|_m = 0.5\cdot 1.5$ V $\to V_{ind} = 1.11\cdot 0.75 = 0.8325$ V.

2.5. $v_s^2 + v_r^2 = v_{sr}^2,\ v_r^2 = (0.75)^2,\ v_{sr}^2 = (6.51)^2$, hence $v_s = \sqrt{(6.51)^2 - (0.75)^2} = 6.47\,\text{V}$.

2.6. (a) a_0 = average value = $\tfrac{1}{2}E$; $a_n = 0$. This applies to all odd functions.

$b_n = \frac{2}{T}\int_0^T x(t)\sin n\omega_0 t dt,\ x(t) = Et/T$ for $0 < t < T$;

$$\int_0^T t\sin n\omega_0 t dt = \left[t\cdot\frac{-1}{n\omega_0}\cos n\omega_0 t\right]_0^T + \int_0^T \frac{1}{n\omega_0}\cos n\omega_0 t dt = \frac{-T}{n\omega_0}\cos n\omega_0 T + 0 = \frac{-T}{n\omega_0}$$

(because $n\omega_0 T = 2\pi n$). So: $b_n = \frac{2E}{T^2}\cdot\frac{-T}{n\omega_0} = \frac{-E}{n\pi}$, hence

$$x(t) = \frac{1}{2}E - \frac{E}{\pi}\left(\sin\omega t + \frac{1}{2}\sin 2\omega t + \frac{1}{3}\sin 3\omega t + \ldots\right).$$

(b) T equals half the period of the sine wave: $x(t) = E\sin(\pi t/T)$ for $0 < t < T$.

$\omega_0 = 2\pi/T = 2\omega$.

$$a_0 = \frac{1}{T}\int_0^T x(t)dt = \frac{E}{T}\int_0^T \sin\frac{\pi}{T}t\,dt = \frac{E}{T}\left[-\frac{T}{\pi}\cos\frac{\pi}{T}t\right]_0^T = \frac{2E}{\pi}.$$

$$a_n = \frac{2}{T}\int_0^T x(t)\cos n\omega_0 t\,dt = \frac{2E}{T}\int_0^T \sin\frac{\pi t}{T}\cos\frac{2\pi nt}{T}dt$$

$$= \frac{2E}{T}\frac{1}{2}\int_0^T\left(\sin\frac{\pi}{T}(1+2n)t + \sin\frac{\pi}{T}(1-2n)t\right)dt$$

$$= \frac{E}{T}\left[-\frac{\cos(\pi/T)(1+2n)t}{(\pi/T)(1+2n)} - \frac{\cos(\pi/\mathrm{T})(1-2n)t}{(\pi/\mathrm{T})(1-2n)}\right]_0^T$$

$$= -\frac{E}{\pi}\left(\frac{\cos(1+2n)\pi - 1}{1+2n} + \frac{\cos(1-2n)\pi - 1}{1-2n}\right)$$

$$= -\frac{E}{\pi}\left(\frac{-2}{1+2n} + \frac{-2}{1-2n}\right) = \frac{4E}{\pi}\cdot\frac{1}{1-4n^2}, \text{ for } n > 0.$$

$$b_n = \frac{2}{T}\int_0^T x(t)\sin n\omega_0 t dt = \frac{2E}{T}\int_0^T \sin\frac{\pi}{\mathrm{T}}t \sin\frac{2\pi n}{T}t dt,$$

from which it follows that: $b_n = 0$ (for all n). This is true in the case of all even functions.

$$x(t) = \frac{2E}{\pi} - \frac{4E}{\pi}\left(\frac{1}{3}\cos 2\omega t + \frac{1}{15}\cos 4\omega t + \frac{1}{35}\cos 6\omega t + \dots\right),\ (n\omega_0 = 2n\omega).$$

2.7. The spectral noise power is $P = 4kT = 4\cdot 1.38\cdot 10^{-23}\cdot 290 = 1.6\cdot 10^{-20}\,\mathrm{W/Hz}$. The bandwidth B = 10 kHz, so $P\cdot B = 1.6\cdot 10^{-16}\mathrm{W}$. This equals V^2/R, with V the rms value of the noise voltage: $V = \sqrt{P\cdot B\cdot R} = \sqrt{1.6\cdot 10^{-16}\cdot 10^4} = 1.26\mu\mathrm{V}$.

Aperiodical signals

2.8. $$a_0 = \frac{1}{T}\int_0^T x(t)dt = \frac{E}{T}\int_0^{T/2}\sin\frac{2\pi}{T}t dt = \frac{E}{\pi}.$$

$$a_n = \frac{2}{T}\int_0^{T/2} E\sin\frac{2\pi}{T}t\cos\frac{2\pi n}{T}t dt$$

$$= \frac{2E}{T}\cdot\frac{1}{2}\int_0^{T/2}\left(\sin\frac{2\pi}{T}(1+n)t + \sin\frac{2\pi}{T}(1-n)t\right)dt$$

$$= \frac{E}{T}\left[-\frac{\cos(2\pi/T)(1+n)t}{(2\pi/T)(1+n)} - \frac{\cos(2\pi/T)(1-n)t}{(2\pi/T)(1-n)}\right]_0^{T/2}$$

$$= -\frac{E}{2\pi}\left(\frac{\cos\pi(1+n)-1}{1+n} + \frac{\cos\pi(1-n)-1}{1-n}\right).$$

With $\cos\pi(1\pm n) = -\cos n\pi$ this can be written as:

$$a_n = \frac{E}{2\pi}\left(\frac{1}{1+n} + \frac{1}{1-n}\right)(\cos n\pi + 1) = \frac{E}{\pi}\frac{(\cos n\pi + 1)}{1-n^2}, \text{ for } n > 0,\ n \neq 1.$$

$$b_n = \frac{2}{T}\int_0^{T/2} E\sin\frac{2\pi}{T}t\sin\frac{2\pi n}{T}t dt = -\frac{2}{T}\cdot\frac{E}{2}\int_0^{T/2}\left(\cos\frac{2\pi}{T}(1+n)t - \cos\frac{2\pi}{T}(1-n)t\right)dt$$

$$= -\frac{E}{T}\left[\frac{\sin(2\pi/T)(1+n)t}{(2\pi/T)(1+n)} - \frac{\sin(2\pi/T)(1-n)t}{(2\pi/T)(1-n)}\right]_0^{T/2}$$

Which is zero for all n, $n \neq 1$.

$$b_1 = \lim_{n\to 1}\frac{E}{T}\left[\frac{\sin(2\pi/T)(1-n)t}{(2\pi/T)(1-n)} - \frac{\sin(2\pi/T)(1+n)t}{(2\pi/T)(1+n)}\right]_0^{T/2}$$

$$= \frac{E}{T}\left[t - \frac{\sin(4\pi/T)t}{(4\pi/T)}\right]_0^{T/2} = \frac{1}{2}E.$$

From a similar calculation it follows that $a_1 = 0$.

With equations (2.8) we find:

$$|C_0| = \frac{E}{2\pi}; |C_1| = \frac{E}{4}; |C_n| = \frac{E}{2\pi}\frac{\cos\pi n + 1}{1-n^2} \text{and } \arg C_n = -\frac{\pi}{2}$$

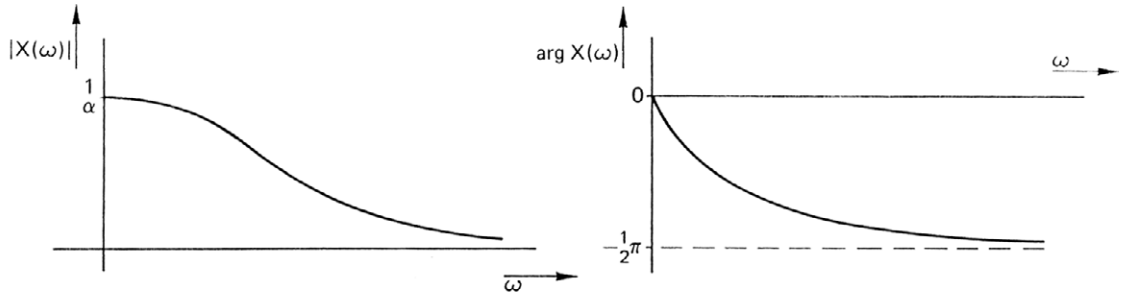
2.9. (a) Apply equation (2.15):

$$\int_{-\infty}^{\infty} |x(t)|\,dt = \int_0^{\infty} e^{-\alpha t} dt = \frac{1}{\alpha}, \text{ hence finite.}$$

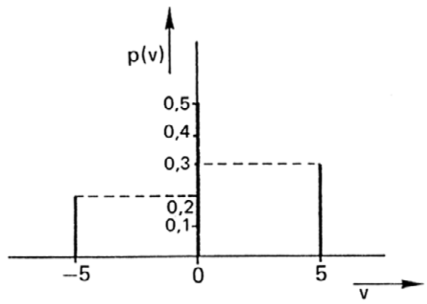
(b) From equation (2.10a) it follows that:

$$X(\omega) = \int_{-\infty}^{\infty} e^{-\alpha t} e^{-j\omega t} dt = \int_0^{\infty} e^{-(\alpha + j\omega)t} dt = \frac{1}{\alpha + j\omega}.$$

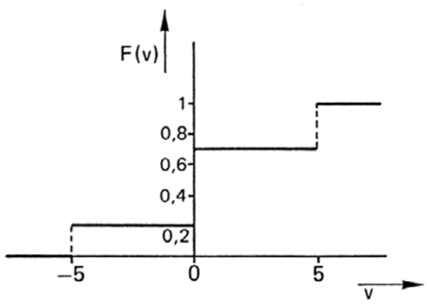
(c) $|X(\omega)| = \dfrac{1}{\sqrt{\alpha^2 + \omega^2}}$; $\arg X(\omega) = -\tan^{-1}\dfrac{\omega}{\alpha}$.



2.10.



probability density function



distribution function

2.11. $E(y) = \int_{-\infty}^{\infty} yp(y)dy = \int_0^{\infty} xp(x)dx$

$$= \int_0^{\infty} x \frac{1}{\sigma\sqrt{2\pi}} e^{-x^2/2\sigma^2} dx = \frac{1}{\sigma\sqrt{2\pi}} \cdot \frac{1}{2} \int_0^{\infty} e^{-x^2/2\sigma^2} dx^2 = \frac{\sigma}{\sqrt{2\pi}}.$$

3. Networks

Electric networks

3.1. a. R_2 in series with R_3: 800 Ω; R_1 in parallel with 800 Ω: 5600//800 = 1/(1/5600 + 1/800) = 700 Ω, corresponding to a single resistance of 700 Ω.

b. Similar to (a): single selfinductance of 3 mH.

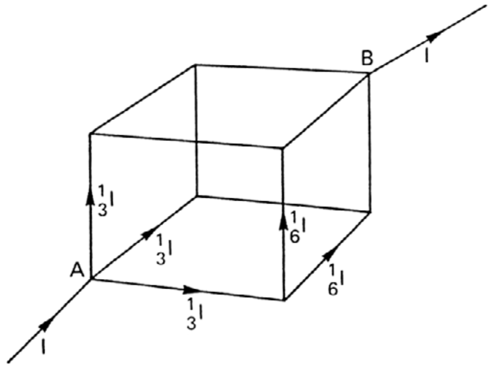
c. C_2 in series with C_3 is equivalent to a capacitance of 235 μF. C_1 in parallel to 235 μF results in a single capacitance of 236 μF.

d. A single voltage source of 4.9 V.

e. A single current source of $1.2 - 125 \cdot 10^{-3} + (-500 \cdot 10^{-6}) = 1.0745$ A.

f. A single resistance of 200 Ω: first, combine R_1 and R_2, then (R_1+R_2) parallel to R_3, then {(R_1+R_2)//R_3} in series with R_4, and so on.

3.2. Due to symmetry, the current splits up into equal parts at each node.



The voltage drop V_{AB} (along an arbitrary shortest path from A to B) is: $V_{AB} = R_{edge}(I/3 + I/6 + I/3) = (5/6)IR_{edge}$; $R_{cube} = V_{AB}/I = (5/6)R_{edge} = 5/6\ \Omega$.

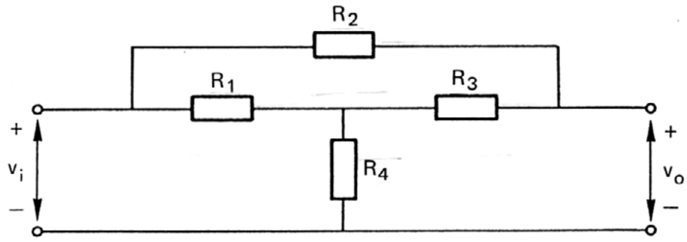
3.3. a. $\dfrac{v_o}{v_i} = \dfrac{R_2}{R_1 + R_2} = \dfrac{1}{101} \approx 0.01$ (voltage divider).

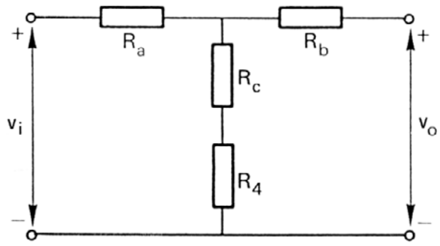
b. $\dfrac{i_o}{i_i} = \dfrac{R_1}{R_1 + R_2} = \dfrac{1}{101} \approx 0.01$ (current divider).

c. $\dfrac{v_o}{v_i} = \dfrac{R_4}{R_3 + R_4} \cdot \dfrac{R_2 // (R_3 + R_4)}{R_1 + R_2 // R_3 + R_4} = 0.45 \cdot \dfrac{36.12}{18 + 36.12} = 0.45 \cdot 0.67 = 0.3$.

(Application of twice the voltage divider rule: once to resistances R_3 and R_4 and once to resistances R_1 and $R_2//(R_3+R_4)$).

3.4. Application of the transformation formula to resistances R_1, R_2 and R_3:





$R_a = \dfrac{R_1 R_2}{R_1 + R_2 + R_3} = \dfrac{2}{3}$ kΩ; $R_b = R_c = R_a = 2/3$ kΩ (because of symmetry);

$\dfrac{v_o}{v_i} = \dfrac{R_c + R_4}{R_a + R_c + R_4} = 0.8$ (voltage divider).

3.5. a. $v_o + RC\dfrac{dv_o}{dt} = v_i$. b. $\dfrac{R}{L}v_o + \dfrac{dv_o}{dt} = \dfrac{dv_i}{dt}$. c. $v_o + LC\dfrac{d^2v_o}{dt^2} = LC\dfrac{d^2v_i}{dt^2}$.

d. $v_o + RC\dfrac{dv_o}{dt} = RC\dfrac{dv_i}{dt}$. e. $v_o + \dfrac{L}{R}\dfrac{dv_o}{dt} = v_i$. f. $v_o + LC\dfrac{d^2v_o}{dt^2} = v_i$.

3.6. $I_c = C\dfrac{dV_c}{dt}$ therefore $\Delta V_c = \dfrac{1}{C}I_c\Delta t + V_c(0) = \dfrac{I_c\Delta t}{C}$. $C = \dfrac{I_c\Delta t}{\Delta V_c} = \dfrac{10^{-6}\cdot 100}{20} = 5\mu$F.

Generalized network elements

3.7. *Generalized capacitances*

- thermal capacitance: $q = C_{th}\dfrac{dT_{ab}}{dt}$ (see page 43)

- mass: $F = C_{mech}\dfrac{dv_{ab}}{dt}$ (see page 44)

- moment of inertia (ratio between torque and angular acceleration): $T_{ab} = J\dfrac{d\Omega_{ab}}{dt}$

Generalized selfinductance

- stiffness: $v_{ab} = \dfrac{1}{K}\dfrac{dF}{dt}$ (see page 44)

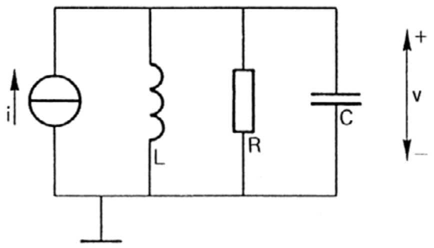
Generalized resistances

- damping: $v_{ab} = \dfrac{1}{b}F$ (see page 44)
- thermal resistance: $T_{ab} = R_{th}q$.

3.8. I-variables: force F; heat flow q and mass flow;
integrated I-variables: electric charge $Q = \int I dt$ and heat (energy) $\int q dt$;
V-variables: angular velocity Ω and temperature difference T;
integrated V-variable: angular displacement $\varphi = \Omega dt$.

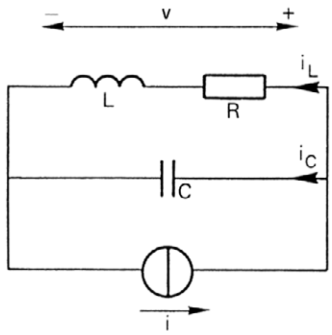
3.9. The equation of motion of the mechanical system is: $F = k\int v\,dt + bv + m\dfrac{dv}{dt}$.

The analog equation of the electrical system is: $i = \dfrac{1}{L}\int v\,dt + \dfrac{1}{R}v + C\dfrac{dv}{dt}$. The corresponding electronic network is given below:



The relation between F and x is: $F = kx + b\dfrac{dx}{dt} + m\dfrac{d^2x}{dt^2}$.

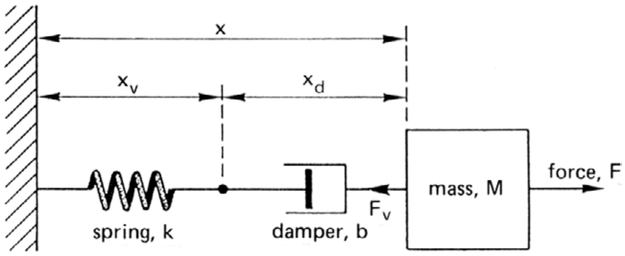
3.10.



The electrical system is described according to the following differential equations:
$v = \dfrac{1}{C}\int i_C dt$, $u = i_L R + L\dfrac{di_L}{dt}$, $i_C + i_L = i$.

Eliminating i_C and i_L results in: $LC\dfrac{d^2v}{dt^2} + RC\dfrac{dv}{dt} + v = L\dfrac{di}{dt} + Ri$.

The selfinductance and the resistance are equivalent with a mechanical spring and damper, and are connected in series. The capacitance is equivalent to a mass. The current source supplies current to the capacitance which in turn is loaded with (supplies current to) the inductance and the resistance. Analogously, the force is applied to the mass, which is loaded by the spring and the damper.



The mechanical system is described according to the differential equations: $m\dfrac{d^2x}{dt^2}=F-F_v$, $F_v=kx_v=b\dfrac{dx_d}{dt}$, $x_v+x_d=x$, $v=\dfrac{dx}{dt}$.

Eliminating F_v, x_v and x_d results in: $\dfrac{m}{k}\dfrac{d^2v}{dt^2}+\dfrac{m}{b}\dfrac{dv}{dt}+v=\dfrac{1}{k}\dfrac{dF}{dt}+\dfrac{F}{b}$, which is equivalent to the electrical system equation.

4. Mathematical tools

Complex variables

4.1. Apply the rules for series and parallel impedances: $Z_s=Z_1+Z_2+\ldots$; $Y_p=Y_1+Y_2+\ldots$ or $1/Z_p=1/Z_1+1/Z_2+\ldots$

a. $R\,//\,\dfrac{1}{j\omega C}=\dfrac{R}{1+j\omega RC}$.

b. $R_2\,//\left(R_1+\dfrac{1}{j\omega C}\right)=\dfrac{R_2(1+j\omega R_1C)}{1+j\omega(R_1+R_2)C}$.

c. $\dfrac{1}{j\omega C_2}\,//\left(R+\dfrac{1}{j\omega C_1}\right)=\dfrac{1+j\omega RC_1}{j\omega(C_1+C_2)-\omega^2C_1C_2R}$.

d. $R+j\omega L+\dfrac{1}{j\omega C}$. This is a series resonance circuit, with $Z=0$ if $R=0$ and $\omega=\dfrac{1}{\sqrt{LC}}$.

e. $\dfrac{1}{Y}$; $Y=\dfrac{1}{R}+j\omega C+\dfrac{1}{j\omega L}\rightarrow\quad Z=\dfrac{R}{1+jR(\omega C-1/\omega L)}$. This is a parallel resonance circuit, with $Y=0$ if $1/R=0$ $(R\rightarrow\infty)$ and $\omega=1/\sqrt{LC}$.

f. $(R+j\omega L)\,//\,\dfrac{1}{j\omega C}=\dfrac{R+j\omega L}{1+j\omega RC-\omega^2LC}$.

4.2. Apply the voltage divider rule to each network: $\dfrac{V_o}{V_i}=\dfrac{Z_2}{Z_1+Z_2}$.

a. $\dfrac{1}{1+j\omega RC}$.

b. $\dfrac{j\omega L}{R+j\omega L}$.

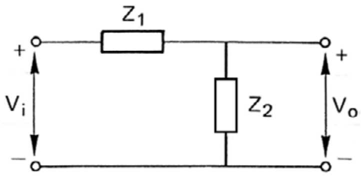
c. $\dfrac{j\omega L}{j\omega L+1/j\omega C}=\dfrac{-\omega^2LC}{1-\omega^2LC}$.

d. $\dfrac{j\omega RC}{1+j\omega RC}$.

e. $\dfrac{R}{R+j\omega L}$.

f. $\dfrac{1/j\omega C}{j\omega L+1/j\omega C}=\dfrac{1}{1-\omega^2 LC}$.

4.3. Simplify the circuit to arrive at the next network:



$Z_1=\dfrac{R_1}{1+j\omega R_1C_1},\ Z_2=\dfrac{R_2}{1+j\omega R_2C_2},$

$\dfrac{V_o}{V_i}=\dfrac{Z_2}{Z_1+Z_2}=\dfrac{R_2(1+j\omega R_1C_1)}{R_1(1+j\omega R_2C_2)+R_2(1+j\omega R_1C_1)}=$

$=\dfrac{R_2}{R_1+R_2}\cdot\dfrac{1+j\omega R_1C_1}{1+j\omega R_p(C_1+C_2)}$, where $R_p=\dfrac{R_1R_2}{R_1+R_2}$.

The transfer does not depend on the frequency if $R_1C_1 = R_p(C_1 + C_2)$, from which follows: $R_1C_1=R_2C_2$.

4.4. $V_a=\dfrac{RV_i}{R+j\omega L}, V_b=\dfrac{V_i}{1+j\omega RC},$

$\dfrac{V_o}{V_i}=\dfrac{V_a-V_b}{V_i}=\dfrac{R}{R+j\omega L}-\dfrac{1}{1+j\omega RC}=\dfrac{1}{1+j\omega L/R}-\dfrac{1}{1+j\omega RC}$.

$V_0/V_i = 0$, irrespective of V_i, if $L/R = RC$ or $L=R^2C$.

Laplace variables

4.5. The rules for series and parallel impedances are also applicable in the Laplace domain.

a. $R\,//\,\dfrac{1}{pC}=\dfrac{R}{1+pRC}$.

b. $\left(R_1+\dfrac{1}{pC}\right)//\,R_2=\dfrac{R_2(1+pR_1C)}{1+p(R_1+R_2)C}$.

c. $\left(R_1+\dfrac{1}{pC_1}\right)//\,\dfrac{1}{pC_2}=\dfrac{1+pRC_1}{p(C_1+C_2)+p^2C_1C_2R^2}$.

d. $R+pL+\dfrac{1}{pC}$.

e. $\left(\dfrac{1}{R}+\dfrac{1}{pL}+pC\right)^{-1}$.

f. $(R+pL)\,//\,\dfrac{1}{pC}=\dfrac{R+pL}{1+pRC+p^2LC}$.

4.6. Replace $j\omega$ with p and $-\omega^2$ with p^2.

a. $\dfrac{1}{1+pRC}$; no zeroes, pole $p = -1/RC$.

b. $\dfrac{pL}{R+pL}$; zero $p = 0$, pole $p = -R/L$.

c. $\dfrac{p^2LC}{1+p^2LC}$; double zero $p = 0$, two poles $p = \dfrac{\pm j}{\sqrt{LC}}$.

d. $\dfrac{pRC}{1+pRC}$; zero $p = 0$, pole $p = -1/RC$.

e. $\dfrac{R}{R+pL}$; no zeroes, pole $p = -R/L$.

f. $\dfrac{1}{1+p^2LC}$; no zeroes, two poles $p = \dfrac{\pm j}{\sqrt{LC}}$.

4.7. a. $\dfrac{V_o}{V_i} = \dfrac{pRC}{1+pRC}$, $V_i(p) = \dfrac{E}{p}$, $V_o(p) = \dfrac{pRC}{1+pRC} \cdot \dfrac{E}{p} = \dfrac{E}{p+1/RC}$.

Inverse transformation: $v_o(t) = E \cdot e^{-t/RC}$, $t > 0$.

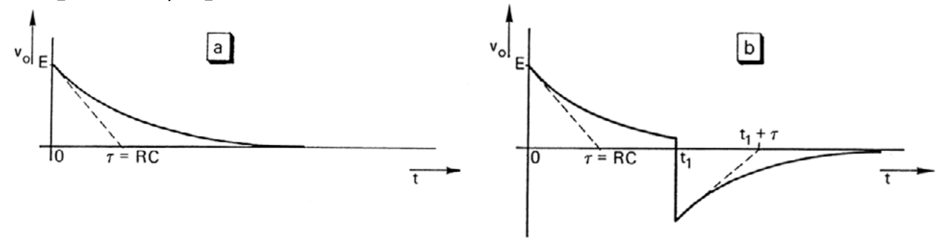
b. The input signal can be seen as being composed of a positive step voltage E at $t = 0$ and a negative step voltage $-E$ at $t = t_1$.

$$V_i(p) = \frac{E}{p} - e^{-pt_1}\frac{E}{p} \;\rightarrow\; V_o(p) = \frac{E}{p+1/RC} - \frac{e^{-pt_1}E}{p+1/RC} \;\rightarrow$$

$v_o(t) = E\,e^{-t/RC}$, $0 \le t < t_1$,

$v_o(t) = E\,e^{-t/RC} - E\,e^{-(t-t_1)/RC}$, $t \ge t_1$

The step at $t = t_1$ equals $-E$ volt.



4.8. The differential equation of the system can be deduced from:

$v_i = iR + L\dfrac{di}{dt} + v_o$ and $i = C\dfrac{dv_o}{dt}$: $v_i = v_o + RC\dfrac{dv_o}{dt} + LC\dfrac{d^2v_0}{dt^2}$

Using rules (4.10) and (4.13), the Laplace equation is:

$V_i = V_o + RC(pV_o - v_o(0)) + LC(p^2V_o - pv_o(0) - v_o'(0))$.

The signal's history (v_i for $t < 0$) is not relevant to the course of v_o for $t \ge 0$, only the boundary conditions are relevant:

$v_o(0) = 0$, $v_o'(t) = \dfrac{i(t)}{C} \;\rightarrow\; v_o'(0) = \dfrac{i_o}{C}$, $v_i = 0$ for $t > 0$.

From this it follows that: $V_o(p) = \dfrac{i_o/C}{p^2 + pR/L + 1/LC}$.

a. For $R_2 > 4L/C$ the denominator of the polynome can be factorized as $(p + a_1)(p + a_2)$, with $a_{1,2} = \dfrac{R}{2L} \pm \sqrt{\dfrac{R^2}{4L^2} - \dfrac{1}{LC}}$, hence:

$$V_o = \frac{i_o/C}{(p+a_1)(p+a_2)} = \frac{i_o}{C}\frac{1}{a_1 - a_2}\left(\frac{-1}{p+a_1} + \frac{1}{p+a_2}\right).$$

The inverse transformation results in:

$$v_o(t) = \frac{i_o}{C}\frac{1}{a_1 - a_2}(-e^{-a_1t} + e^{a_2t}),\; t > 0.$$

b. In the case of $R^2 < 4L/C$ this factorization is not possible. V_o can be rewritten as:

$$V_o = \frac{i_o/C}{(p+a)^2 + \omega^2},$$

(one of the terms in Table 4.1, with $a = R/2L$ and $\omega = 1/LC - R^2/4L^2$.)
Inverse transformation gives:

$$v_o(t) = \frac{i_o}{\omega C} e^{-at} \sin \omega t, \ t > 0.$$

c. For $R^2 = 4\frac{L}{C}$, $\omega^2 = 0$, hence $V_o = \dfrac{i_o/C}{(p+a)^2}$

Using the transformation $1/p^2 \leftrightarrow t$ and Equation (4.8), this results in:

$$v_o(t) = \frac{i_o}{C} t e^{-at}, \ t > 0.$$

Conclusion:
a. $R = 400\Omega$: $v_o(t)$ is an exponentially decaying voltage.
b. $R = 120\Omega$: $v_o(t)$ is a sinusoidal voltage with exponentially decaying amplitude.
c. $R = 200\Omega$: $v_o(t)$ first increases with time, and then decreases exponentially down to zero.

5. Models

System models

5.1. The source resistance is found by disregarding all sources; the source resistance is the equivalent resistance of the resultant two-pole network.

a. $V_o = \dfrac{R_2}{R_1 + R_2} = 6\text{V}$; $R_s = R_3 + R_1//R_2 = 10.025\text{k}\Omega$.

b. $V_o = IR_1 = 90$ V; $R_s = R_1 + R_2 + R_3 = 90\Omega$.

c. Apply the principle of superposition with respect to the two current sources and the rule for the current divider. The result will be:

$$V_o = I_1\left(\frac{R_1 R_2}{R_1 + R_2 + R_3}\right) + I_2(R_2 //(R_1 + R_3)) \approx 34\text{V} + 50.4\text{V} = 84.4\text{V};$$

$R_s = R_2 //(R_1 + R_3) \approx 25.2$ kΩ.

5.2. CMRR(DC) = –90 dB, 20 log $\dfrac{V_o}{V_i}$ = –90, $\dfrac{V_o}{V_i}$ = $10^{-4.5} \approx 32$ μV/V

CMRR(1 kHz) = –80 dB, 20 log $\dfrac{V_o}{V_i}$ = –80, $\dfrac{V_o}{V_i}$ = $10^{-4} \approx 100$ μV/V

SVRR = –110 dB, 20 log $\dfrac{V_o}{V_i}$ = –110, $\dfrac{V_o}{V_i}$ = $10^{-5.5} \approx 3.2$ μV/V

5.3. $I_m = \dfrac{R_s}{R_L + R_s} I_s \approx \left(1 - \dfrac{R_L}{R_s}\right) I_s \rightarrow$ relative error $\approx \dfrac{-R_L}{R_s} < 0.5\% \rightarrow R_L \leq 500\ \Omega$.

5.4. (a) $V_s \approx 9.6$ V; $\dfrac{R_L}{R_s + R_L} \cdot 9.6 = 8.0\text{V} \rightarrow R_s = 2\text{k}\Omega$

(b) $V_o = \dfrac{R_i}{R_s + R_i} \cdot V_s \approx \left(1 - \dfrac{R_s}{R_i}\right) V_s \rightarrow$ the maximum relative error is $-\dfrac{R_s}{R_i} = -\dfrac{2 \cdot 10^3}{10^7} = -0.02\%$.

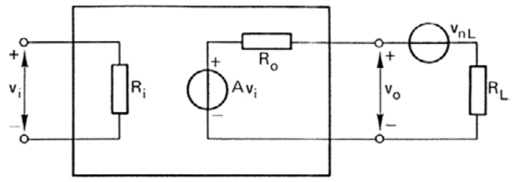
5.5. See Section 5.1.3.

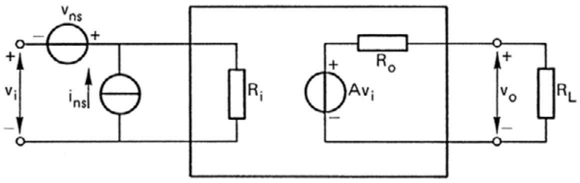
5.6. $A_v = \frac{v_o}{v_i} = \frac{AR_L}{R_o + R_L}$; $i_o = \frac{Av_i}{R_o + R_L} = \frac{AR_i i_i}{R_o + R_L} \rightarrow A_i = \frac{i_o}{i_i} = \frac{AR_i}{R_o + R_L}$;

$$A_p = \frac{P_o}{P_i} = \frac{v_o i_o}{v_i i_i} = A_v A_i = \frac{A^2 R_i R_L}{(R_o + R_L)^2}.$$

Signal models

5.7. The next two systems must be equivalent:





As a criterion for equivalence, take v_o:

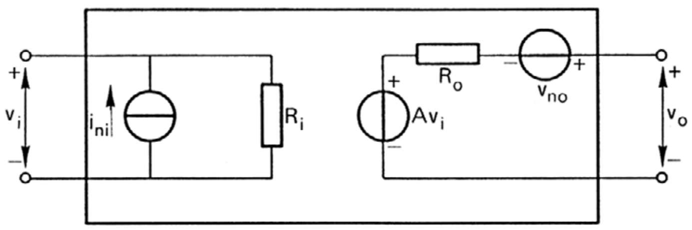
With short-circuited inputs:

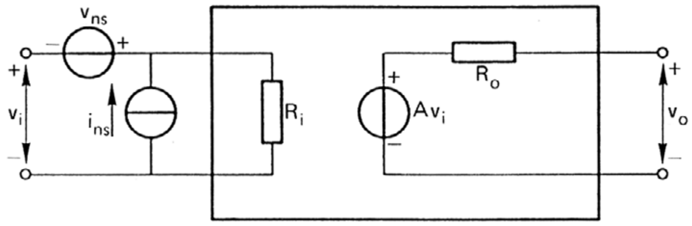
$$v_o = v_{nL} \frac{R_o}{R_o + R_L} = Av_{ns} \frac{R_L}{R_o + R_L} \rightarrow v_{ns} = \frac{R_o v_{nL}}{R_L A}.$$

With open inputs:

$$v_o = v_{nL} \frac{R_o}{R_o + R_L} = Ai_{ns} R_i \frac{R_L}{R_o + R_L} \rightarrow i_{ns} = \frac{R_o v_{nL}}{R_L R_i A}.$$

5.8. In the preceding exercise, the equivalent noise sources depend on R_L, as part of the output voltage divider. In this exercise, all noise sources originate from the system itself, so the result does not depend on R_L. Therefore, we can put $R_L \rightarrow \infty$ for the sake of simplicity.





Short-circuited input terminals:

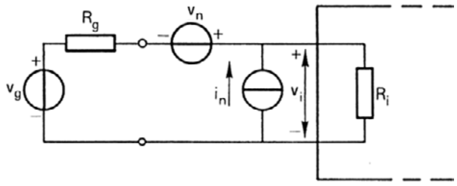
$$v_o = v_{no} = Av_{ns} \rightarrow v_{ns} = \frac{v_{no}}{A}.$$

Open input terminals:

$$v_o^2 = (Ai_{ni}R_i)^2 + v_{no}^2 = (Ai_{ns}R_i)^2 \rightarrow i_{ns}^2 = i_{ni}^2 + \left(\frac{v_{no}}{AR_i}\right)^2.$$

5.9. The output offset voltage is $A(V_n + I_n R_g)$. At 20 °C, V_n= 1 mV and I_n = 10 nA, hence $V_{\text{off,o}} = 10(10^{-3} + 10^{-8}\cdot 10^4) = 11$ mV.
The maximum offset occurs at T = 50 °C. At that temperature, the offset is V_n = 1 mV + 30·10 μV = 1.3 mV and I_n = 10 nA · 2^3 = 80 nA, thus $V_{\text{off,o}} = 10(1.3\cdot 10^{-3} + 80\cdot 10^{-9}\cdot 10^4) = 21$ mV.

5.10. Make a model of the system:



The signal power P_g towards the system is v_i^2 / R_i, hence

$$P_g = \left(\frac{R_i}{R_i + R_g} v_g\right)^2 \cdot \frac{1}{R_i}$$

Both v_n and i_n contribute to the noise power P_n:

$$P_n = \left(\frac{R_i}{R_i + R_g} v_n\right)^2 \cdot \frac{1}{R_i} + \left(\frac{R_g R_i}{R_g + R_i} i_n\right)^2 \cdot \frac{1}{R_i}$$

$$S/R = \frac{P_g}{P_n} = \frac{v_g^2}{v_n^2 + R_g^2 i_n^2},$$

which is independent of R_i.

6. Frequency diagrams

Bode plots

6.1. a. $H = \dfrac{j\omega R_2 C}{1 + j\omega(R_1 + R_2)C}$; $R_2 C = 0.5$ ms; $(R_1 + R_2)C = 1$ ms

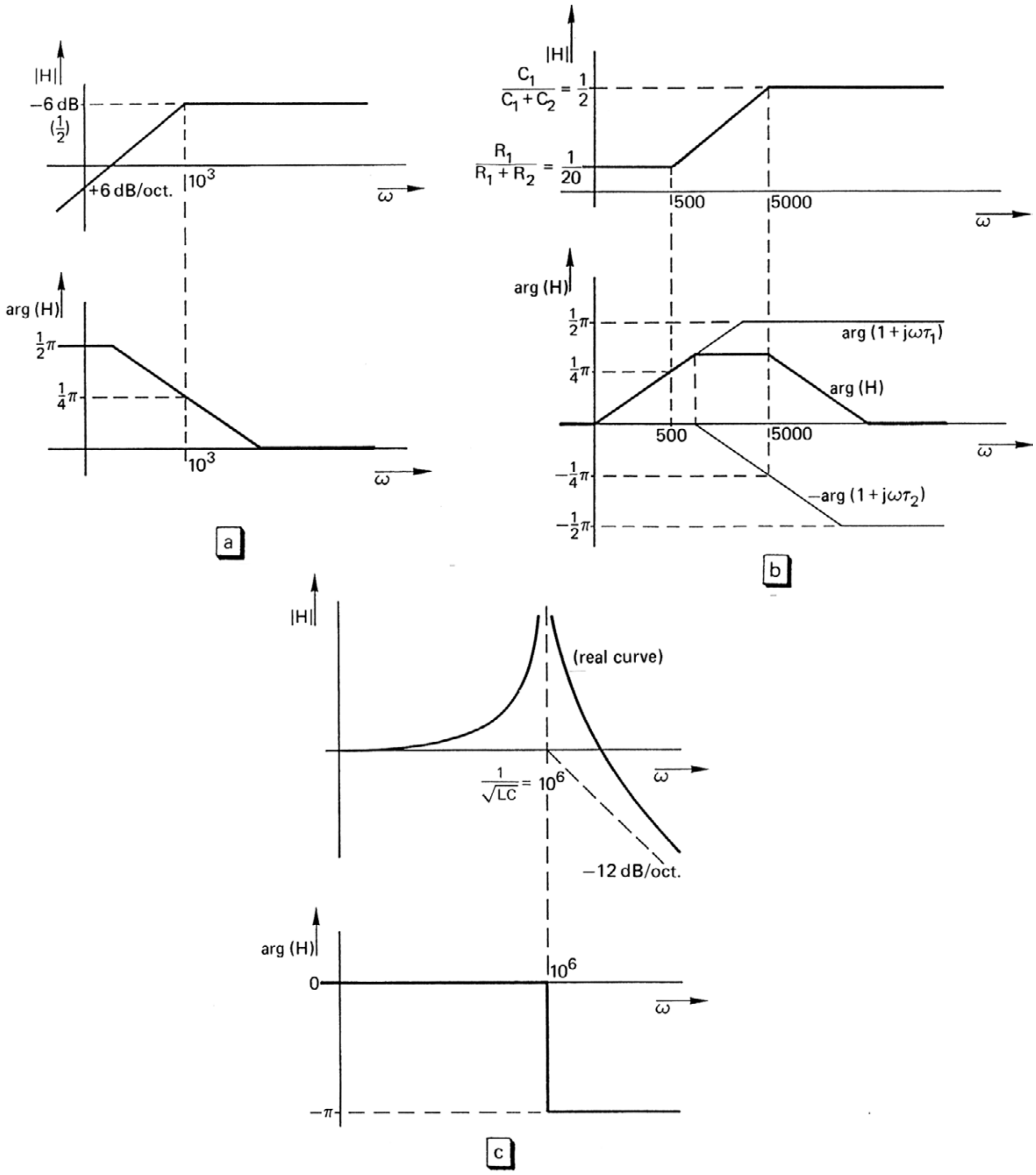
$\omega \rightarrow 0$	$H \rightarrow j\omega R_2 C$	$\lvert H\rvert \rightarrow \omega R_2 C$	H is positive imaginary, so arg $H = \pi/2$
$\omega \rightarrow \infty$	$H \rightarrow \dfrac{R_2}{R_1 + R_2} = \frac{1}{2}$	$\lvert H \rvert \mapsto \dfrac{1}{2} \equiv -6\text{dB}$	arg $H \rightarrow 0$.
$\omega = \omega_k$	$H = \dfrac{jR_2/(R_1 + R_2)}{1 + j}$	$\lvert H \rvert = \dfrac{1}{2\sqrt{2}}$	$\arg H = \dfrac{\pi}{2} - \dfrac{\pi}{4} = \dfrac{\pi}{4}$

Break point: $\omega(R_1 + R_2)C = 1$, or $\omega_k = 10^3$ rad/s.

b. $H = \dfrac{R_2}{R_1 + R_2} \cdot \dfrac{1 + j\omega\tau_1}{1 + j\omega\tau_2}$; $\tau_1 = R_1 C_1 = 2$ ms;

$\tau_2 = \dfrac{R_1 R_2}{R_1 + R_2} \cdot (C_1 + C_2) \approx 0.2$ ms; $\dfrac{R_2}{R_1 + R_2} \approx \dfrac{1}{20}$; (see Exercise 4.3).

The Bode plot can be composed of the diagrams of $R_2/(R_1 + R_2)$ (frequency independent), $1 + j\omega\tau_1$ and $1/(1 + j\omega\tau_2)$.



c. $H = \dfrac{1}{1-\omega^2 LC}$; see also Exercise 3.5f.

$\omega \to 0;\qquad H \to 1 \qquad |H| \to 1 \qquad \arg H \to 0.$

$\omega \to \infty;\qquad H \to \dfrac{-1}{\omega^2 LC} \qquad |H| \to \dfrac{1}{\omega^2 LC} \qquad \arg H \to -\pi.$

$\omega \to \sqrt{\dfrac{1}{LC}};\quad H \to \infty,$ phase indeterminate.

6.2. $$H = \frac{1}{1 + j\omega RC - \omega^2 LC} = \frac{1}{1 + j2z\omega/\omega_0 - \omega^2/\omega_0^2}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 10^7 \text{ rad/s}$$

$$\frac{2z}{\omega_0} = RC \to z = 0.5$$

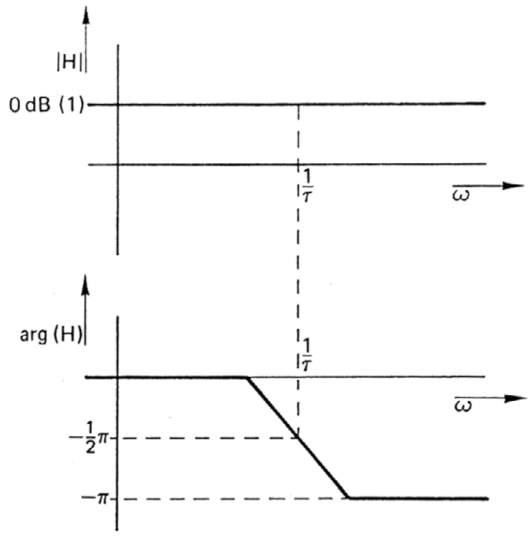
$$\omega_m = \omega_0\sqrt{1-2z^2} = \tfrac{1}{2}\sqrt{2\cdot 10^7}\ \text{ rad / s (see Section 6.1.2)}$$

$$\left|H(\omega_m)\right| = \frac{1}{2z\sqrt{1-z^2}} = \tfrac{2}{3}\sqrt{3}\ \text{ (see Section 6.1.2)}$$

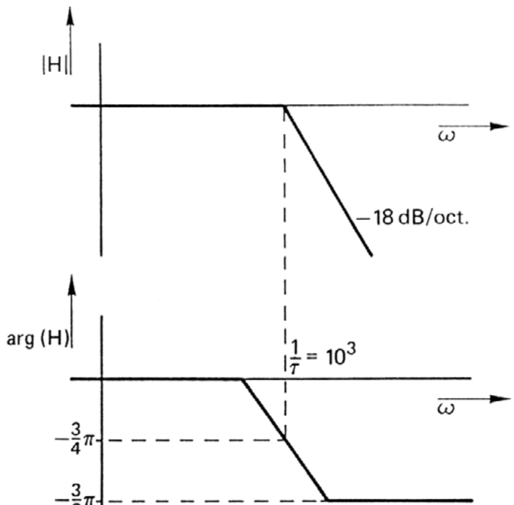
$$\left|H(\omega_m)\right| = \frac{1}{2z} = 1.$$

6.3. $|H| = 1$; $\arg H = -2\tan^{-1}\omega\tau$; $\tau = RC$.

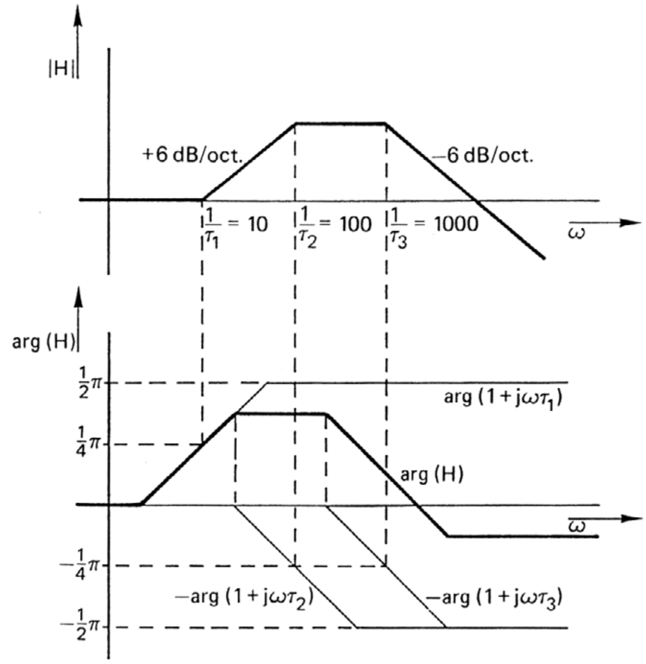
$\omega = 0$: $\arg H = 0$; $\omega = 1/\tau$: $\arg H = -\pi/2$; $\omega \to \infty$: $\arg H = -\pi$ (see the figure below)



6.4. Summing three Bode plots representing the function $1/(1 + j\omega\tau)$.

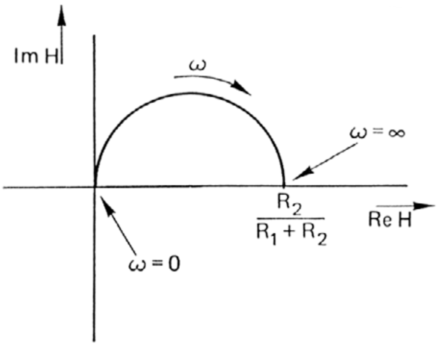


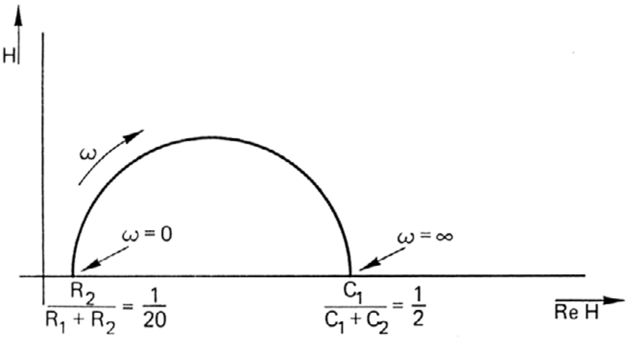
6.5. Summing the Bode plots for the functions $H_1 = 1 + j\omega\tau_1$, $H_2 = 1 + j\omega\tau_2$. and $H_3 = 1 + j\omega\tau_3$.

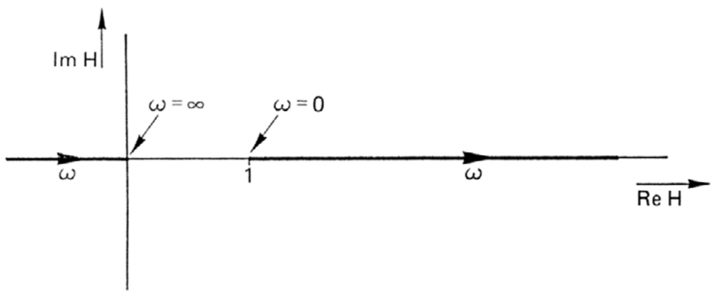


Polar plots

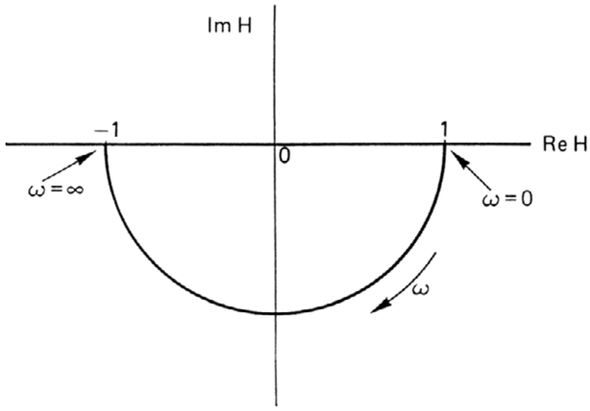
6.6. a. The polar plot is a circle, with starting point 0 and ending point $R_2/(R_1 + R_2)$ (both on the real axis).
The tangent at the starting point makes an angle $\frac{1}{2}\cdot(1 - 0)\cdot\pi$, so that the semi-circle lies in the upper part of the complex plane.
b. The polar plot is a circle. The starting point is $R_2/(R_1 + R_2) \approx \frac{1}{20}$ and the ending point is $R_2/(R_1 + R_2) \cdot (\tau_1/\tau_2) \approx \frac{1}{2} = C_1/(C_1 + C_2)$
As the polar plot turns clockwise, it describes of a semi-circle in the upper half plane. If $R_2/(R_1 + R_2) > C_1/(C_1 + C_2)$, the semi-circle lies in the lower half plane.
c. The function is real for all ω, so the polar plot lies on the real axis. The starting point is 1, the ending point is 0, and the tangent at the ending point makes an angle of $\pi + (0 - 2)\cdot(\pi/2) = 0$.



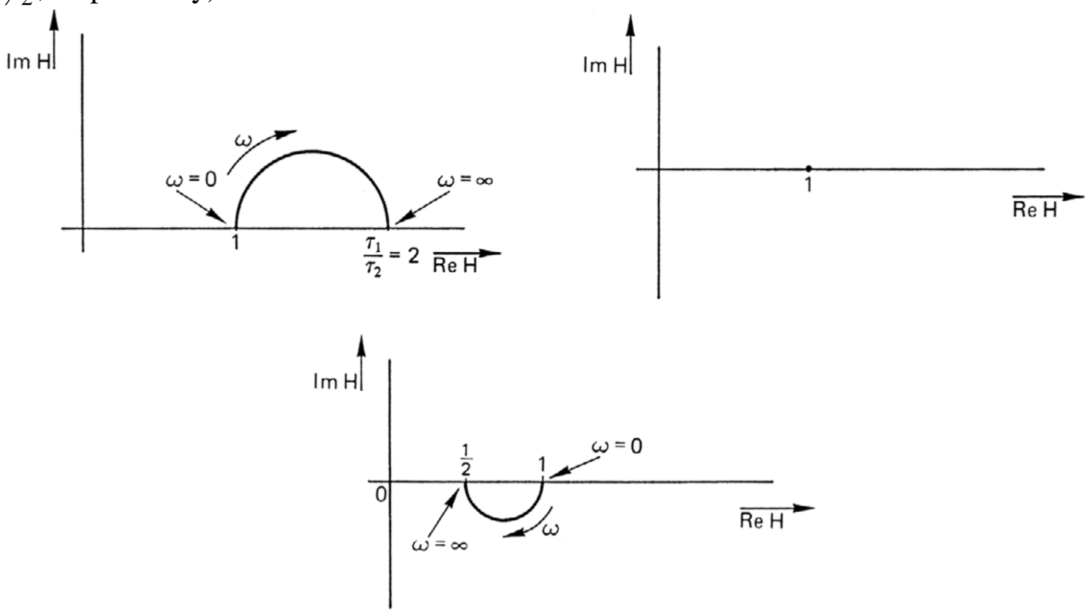




6.7. The network is called an all-pass network because the transfer is the same for all frequencies and it is equal to 1; the polar plot lies on the unity circle. The starting point is 1, the ending point is –1 and the polar plot turns in a clockwise direction.



6.8. The polar plot is a semi-circle, with starting point 1 and ending point τ_1/τ_2 (2, 1 and $\frac{1}{2}$, respectively).



7. Passive electronic components

Passive circuit components

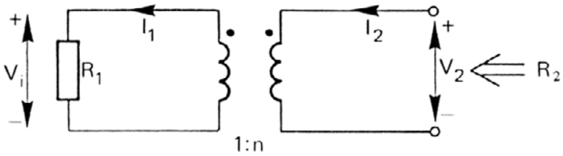
7.1. $i = C\dfrac{dv}{dt}$; $q = Cv$.

7.2. δ is a measure of the dielectric losses in a capacitor; $\tan\delta = I_R/I_C = 1/\omega RC$. (see Section 7.1.2).

7.3. $v = L\dfrac{dv}{dt}$; $\Phi = Li$.

7.4. $B = \mu_0\mu_r H$; $\Phi = \iint \overline{B}\,d\overline{A}$; $[B]$ = T = V s/m^2; $[H]$ = A/m; $[\Phi]$ = Wb = V s.

7.5.



$$R_2 = \frac{V_2}{I_2};\ R_1 = \frac{V_1}{I_1}.$$

$$\frac{V_2}{V_1} = n,\ \frac{I_2}{I_1} = \frac{1}{n} \to R_2 = n^2 \frac{V_1}{I_1} = n^2 R_1.$$

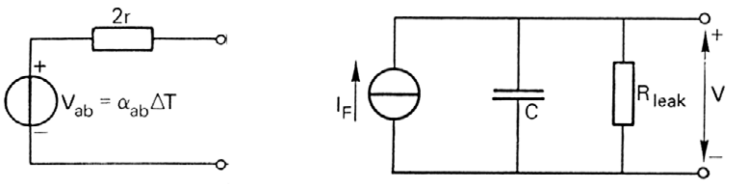
Sensor components

7.6. The absolute error is $R_0\beta T^2$; its maximum value (at T = 100 °C) is –0.58 Ω. For T = 100 °C is, in the case of a linear characteristic, R =100(1 + 3.9·10^{-3}·10^2) = 139 Ω. The non-linearity error is –0.58/139·100% = –0.42%, or –0.58/0.39 = –1.49 °C (see Exercise 1.9).

7.7. For a Pt-100, 0.39 Ω is equivalent to 1 K, so 0.039 Ω is equivalent to 0.1 K. The resistance of each wire is r, so the wire resistance per sensor is $2r$: $2r$ < 0.039 Ω, or r < 20 mΩ.

7.8. The sensitivity of the thermocouple is 40 μV/K (see Table 7.3): 0.5 K ↔ 20 μV and this is just the maximum allowable uncertainty in the offset voltage.

7.9. The common-mode voltage is $\frac{1}{2}v_i$, the differential-mode voltage is $(\Delta R/R)v_i$, hence the smallest voltage that has to be measured is $10^{-6}v_i$. The maximum allowable output voltage due to a common-mode voltage is equal to the output voltage produced by an input differential voltage of $10^{-6}v_i$. So: CMRR > $\frac{1}{2}v_i/10^{-6}v_i = 5\cdot10^5$.

7.10 

I_F is proportional to the derivative of the force on the sensor. The charge produced is $Q = S_qF$, and S_q is the force sensitivity; $I_F = dQ/dt$.

7.11 The slider of the potentiometer divides resistance R into two parts: αR and $(1 - \alpha)R$. The voltage transfer is:

$$\frac{v_o}{v_i} = \frac{R_L // \alpha R}{R_L // \alpha R + (1-\alpha)R} = \frac{\alpha}{1 + (1-\alpha)\alpha R / R_L} \approx \alpha\left(1 - (1-\alpha)\alpha\frac{R}{R_L}\right)$$

The term $(1 - \alpha)\alpha R/R_L$ causes non-linearity (with respect to the transfer $v_o/v_i = \alpha$). This term has a maximum for α = $\frac{1}{2}$; which is why the non-linearity is approximately $R/4R_L$ = 800/(4·10^3) = 2·10^{-3}.

8. Passive filters

First and second order RC-filters

8.1. $|H| = 1/\sqrt{1+\omega^2\tau^2}$, arg $H = -\tan^{-1}\omega\tau$, $\tau = 10^{-3}$ s.

a. $\omega^2\tau^2 = 0.01$ → $|H| = 1$ arg H = –0.10 rad;

b. $\omega^2\tau^2 = 1$ → $|H| = \frac{1}{2}\sqrt{2}$ (–3 dB frequency or break point), arg H = –0.79 rad.

c. $\omega^2\tau^2 = 100$ → $|H| \approx \frac{1}{10}$ arg H = –1.47 rad.

8.2. $|H| = \omega\tau/\sqrt{1+\omega^2\tau^2}$, with $\tau = 0.1$ s.

a. For $\omega^2\tau^2 \gg 1$ $|H| \approx 1$; $\omega \gg 1/\tau$; or $\omega \gg 10$ rad/s;

b. $|H| \approx 0.1$ for $\omega\tau = 0.1$, or $\omega = 1$ rad/s;

c. $|H| \approx 0.01$ for $\omega\tau = 0.01$, or $\omega = 0.1$ rad/s.

8.3. The modulus of the transfer is $|H| = 1/\sqrt{1+\omega^2\tau^2}$.
For the measurement signal, this must be at least 0.97 (namely 3% less than 1). Hence: $1 + \omega^2\tau^2 \le (1/0.97)^2$, or $\omega\tau \le 0.25$.

The largest attenuation occurs at the highest signal frequency (1 Hz = 2π rad/s), so $\tau \le 0.25/2\pi$ s.

The interference signal should be attenuated by at least a factor of 100, so $\omega\tau \ge 100$ (ω is the frequency of the interference signal):

$$\tau \ge \frac{100}{2\pi \cdot 2\text{kHz}} = \frac{0.05}{2\pi} \text{ s.}$$

The limits for the time constants are:

$$\frac{0.05}{2\pi} \le \tau \le \frac{0.25}{2\pi} \text{s.}$$

8.4. The third-harmonic distortion of a triangular signal is 1/9. The slope of a low-pass order n filter is 2^n per octave (a factor of 2 in frequency), or 3^n for a factor of 3 in frequency.

a. The attenuation of the third-harmonic relative to the fundamental is 3 (for a first-order filter), so d_3 (the third-harmonic distortion) after filtering equals $\frac{1}{9} \cdot \frac{1}{3} = \frac{1}{27} \approx 3.7\%$;

b. The attenuation is $(\frac{1}{3})^2$: $d_3 = \frac{1}{9} \cdot (\frac{1}{3})^2 = \frac{1}{81} \approx 1.23\%$;

c. The attenuation is $(\frac{1}{3})^3$: $d_3 = \frac{1}{9} \cdot (\frac{1}{3})^3 = \frac{1}{243} \approx 0.41\%$.

8.5. The frequency ratio between the measurement signal and the interference signal is 5. A low-pass order n filter attenuates the signal with the highest frequency by a factor of 5^n more than the signal with the lower frequency (see also Exercise 8.4). Requirement $5^n \ge 100$, so $n \ge 3$.

Filters of higher order

8.6. The order can be found by counting the number of reactive elements (self-inductances, capacitances) of the circuit in its most simple form. The filter type is found by the determination of the transfer for $\omega \to 0$ and $\omega \to \infty$.

a. Fourth-order low-pass filter;

b. third-order band-pass filter;

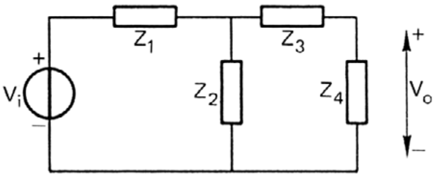
c. third-order high-pass filter.

8.7. $$|H| = \frac{1}{\sqrt{1+(\frac{1}{2})^{2n}}}.$$

a. $|H| = \dfrac{1}{\sqrt{1+\frac{1}{16}}} \approx 0.970 \approx -0.263\text{dB}$;

b. $|H| = \dfrac{1}{\sqrt{1+\frac{1}{64}}} \approx 0.992 \approx -0.070\text{dB}$.

8.8. The transfer function of a network with the structure as shown below is:



$$H = \frac{V_o}{V_i} = \frac{Z_2 Z_4}{Z_1 Z_2 + (Z_1 + Z_2)(Z_3 + Z_4)}.$$

Application of this formula to the given network results in:

$$H = \left[\frac{R_g}{R_L} + 1 + j\omega\left(R_g C_1 + R_g C_2 + \frac{L}{R_L} \right) + (j\omega)^2 \left(LC_2 + LC_1 \frac{R_g}{R_L} \right) + (j\omega)^2 R_g C_1 C_2 L \right]^{-1}$$

Substitution of the numerical values produces:

$$H = \frac{1}{2 + 4j\omega + 4(j\omega)^2 + 2(j\omega)^3}, \text{ so}$$

$$|H| = \frac{1}{\left[(2 - 4\omega^2)^2 + \omega^2 (4 - 2\omega^2)^2 \right]^{1/2}} = \frac{1}{2\sqrt{1 + \omega^6}}$$

As $|H|$ can be written as $(1 + \omega^{2n})^{-1/2}$, with $n = 3$, this is an order 3 Butterworth filter.

8.9. $$H = \frac{1}{1 + j\omega L / R - \omega^2 LC}; \quad |H|^2 = \frac{1}{1 - 2\omega^2 LC + \omega^4 L^2 C^2 + \omega^2 L^2 / R^2}.$$

This is a Butterworth filter if the term with ω^2 is zero, hence $L = 2R^2C$.

In that case,

$$|H|^2 = \frac{1}{1 + 2\omega^4 L^2 C^2} = \frac{1}{1 + (\omega / \omega_c)^{2n}}, \text{ where } \omega_c = 1/\sqrt{LC} \text{ and } n = 2;$$

$$C = \frac{1}{\omega_c^2 L} = 10^{-7}\text{F}; \; R = \sqrt{\frac{L}{2C}} = 50\sqrt{2}\,\Omega$$

9. PN diodes

Properties of pn diodes

9.1. $I = I_o(e^{qV/kT} - 1) \approx I_o e^{qV/kT}$

9.2. $r_d = kT/qI$, g (conductance) $= 1/r_d$. Using the rule of thumb of: $r_d = 25\ \Omega$ at $I = 1$ mA and $T = 300$ K, it follows that:

at $I = 1$ mA; $r_d = 25\ \Omega$, $g = 40$ mA/V

at $I = \frac{1}{2}$ mA; $r_d = 50\ \Omega$, $g = 20$ mA/V

at $I = 1\ \mu$A; $r_d = 25$ kΩ, $g = 40\ \mu$A/V

9.3. The temperature coefficient is about –2.5 mV/K: a temperature increase of 10 K lowers the diode voltage by 25 mV.

9.4. $$V = \frac{kT}{q} \ln \frac{I}{I_o} = kT \frac{\log I / I_o}{\log e} \approx \frac{0.025}{0.434} \log \frac{I}{I_o} = 0.058 \log \frac{I}{I_o}.$$

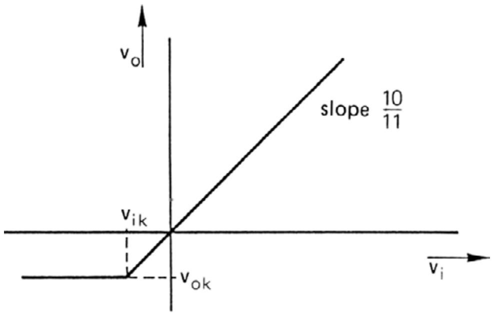
Tenfolding I results in a voltage increase of about 58 mV.

9.5. At 0.1 mA, r_s can be neglected with respect to r_d, but not however at 10 mA. Using the result of Exercise 9.4, the diode voltage V would be 600 + 58 + 58 = 716 mV, if r_s is zero. The measured value is 735 mV, so there is a voltage of 19 mV over r_s. Hence: r_s = 19 mV/10 mA = 1.9 Ω. At 0.1 mA, r_d is about 250 Ω, so the assumption that $r_s \ll r_d$ appears to be valid.

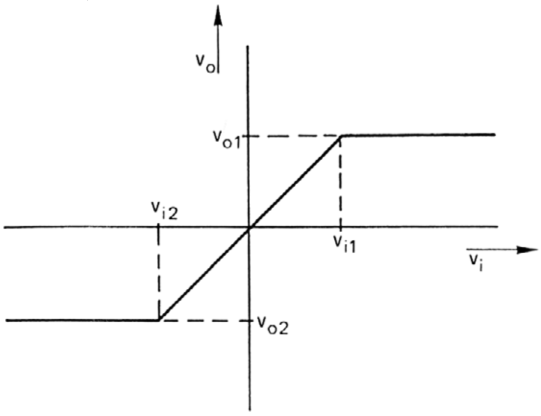
Circuits with pn-diodes

9.6. The output voltage is limited either to V_k or to the Zener voltage V_z. The input voltage is derived from the output voltage.

a.



$v_{ok} = -0.5$ V;
$v_o = (1000/1100)v_i$ as long as the diode is forward biased;
$v_{ik} = (11/10)v_{ok} = -0.55$ V.
b. The same shape; $v_{ok} = 5.5 - 0.5 = 5.0$ V;
$v_{ik} = (11/10)v_{ok} = 5.5$ V;
$v_{o1} = 5.5$ V;
$v_{i1} = (11/10)v_{o1} = 6.05$ V;
$v_{o2} = -0.5$ V;
$v_{i2} = -0.55$ V;
c.



d. The same shape; $v_{o1} = v_{z1} + v_{k2} = 6.5$ V;
$v_{i1} = 7.15$ V;
$v_{o2} = -v_{z2} - v_{k1} = -10.5$ V;
$v_{i2} = -11.55$ V;

9.7. The circuit is sensitive to the negative peak value. For any input voltage, $v_o - v_i \leq V_k$. The output voltage is $v_o = v_{i,min} + V_k$, with values –5.4 V, –0.9 V and 0 V, respectively (in the latter case, the diode remains reverse biased).

9.8. The ripple voltage Δv satisfies the equation $\Delta v / (\hat{v}_i - V_k) \approx T / \tau$ (see Eq. (9.2)). For relatively large signal amplitude, V_k can be ignored in comparison with to the amplitude, hence: $\dfrac{\Delta v}{\hat{v}_i} \approx \dfrac{T}{\tau} = 1\%$; $\tau = 100T = 100 \cdot 0.2\,\text{ms} = 20\,\text{ms}$; $R = \dfrac{\tau}{C} = 200\,\text{k}\Omega$

9.9. The capacitance values are not of relevance here.
a. v_o is a sine wave with an amplitude of 5 V. The clamping level is –0.6 V for the negative peaks, hence $v_{o,av} = -0.6 + 5 = 4.4$ V.
b. The same as (a); the clamping level is +5.4 – 0.6 = 4.8 V, hence $v_{o,av} = 9.8$ V.

c. C_1 and D_1 form a clamping circuit, C_2 and D_2 a peak detector for negative peak values. The output voltage of the clamping part is a sine wave with amplitude 5 V, the positive peaks of which are clamped at +0.6 V. Its negative peak is 0.6 – 10 = –9.4 V, so: $V_o = -9.4 + 0.6 = -8.8$ V.

9.10 For a proper operation the capacitor must be able to charge, so the diode should (at least once) be forward biased. This happens as soon as $\hat{v}_i > V + V_k$, so for any value of $\hat{v}_i$ the voltage V must satisfy $V < \hat{v}_i - V_k$. The clamping level is $V + V_k$ and there is no condition for the minimum value of V.

9.11 The bridge (without a capacitor) acts as a double-sided rectifier, not as a peak detector.

a. The output is a double-sided rectified sine wave.

$\hat{v}_i = 10\sqrt{2}\,\text{V};\ \hat{v}_o = \hat{v}_i - 2V_k \approx 10\sqrt{2} - 1.2 \approx 12.9\,\text{V}.$

This is equal to the ripple voltage

b. $V_o = |V_i| - 2V_k \approx 8.8\,\text{V}.$

This is a DC voltage, so there is no ripple.

c. Just as in (b), the current flows through both other diodes.

9.12 When loaded with a capacitor, the bridge acts as a peak detector.

a. $V_{o,m} \approx \hat{v}_i - 2V_k = 10\sqrt{2} - 1.2 \approx 12.9\,\text{V}.$

$\Delta v = (\hat{v}_i - 2V_k)\cdot(T/\tau)$ with $T = 10$ ms (double frequency) and $\tau = RC = 30$ ms. Hence:

$\Delta v \approx (10/30)\cdot 12.9 = 4.3$ V.

b. At a DC input voltage, the capacitor is charged up to $V_i - 2V_k$; the output voltage is 8.8 V (without ripple).

c. Just as in (b).

9.13 The current through the Zener diode should be more than zero, to guarantee a proper Zener voltage. Hence:

$$I_{R1} \geq I_{RL} \rightarrow \frac{18-5.6}{R_1} \geq \frac{5.6}{R_L} \rightarrow R_L \geq \frac{5.6\cdot 1800}{18-5.6} = 813\Omega.$$

10. Bipolar transistors

Properties of bipolar transistors

10.1. $I_C = I_0(e^{qv_{BE}/kT} - 1) \approx I_0 e^{qv_{BE}/kT}$.

The base-emitter junction must be forward biased, the collector-base junction must be reverse biased if it is to operate properly as a linear amplifying device. For an npn transistor: $V_{BE} \approx 0.6$ V and $V_{BC} \leq 0$ V.

10.2. $I_E = I_B + I_C$ and $I_C = \beta I_B$, hence $I_E = (1 + \beta)I_B = 800\ \mu$A $\rightarrow$

$I_B = \dfrac{800\,\mu\text{A}}{1+200} \approx 4\ \mu$A; $I_C = I_E - I_B = 796\ \mu$A.

10.3. The Early effect in a bipolar transistor accounts for the effect of V_C on I_C; ideally, this effect is zero.

10.4. $V_B = \dfrac{10}{10+15}\cdot 25 = 10$ V $\rightarrow V_E = 10 - 0.6 = 9.4$ V;

$I_E = \dfrac{9.4\,\text{V}}{4700\,\Omega} = 2$ mA; $I_C \approx I_E = 2$ mA;

$V_o = 25$ V – 2 mA·5600 Ω = 13.8 V.

10.5. $V_E(T_1) = -0.6\text{ V} \rightarrow I_{E1} = \dfrac{-0.6\text{V} - (-10\text{V})}{4700\Omega} = 2\text{ mA}$, $I_{C1} \approx I_{E1} = 2\text{ mA}$;

$V_E(T_2) = +0.6\text{ V} \rightarrow I_T = \dfrac{10\text{V} - 0.6\text{V}}{1880\Omega} = 5\text{ mA}$;

$I_T = I_{C1} + I_{E2}$ (Kirchhoff's law) $\rightarrow I_{E2} = 5 - 2 = 3\text{ mA}$, $I_{C2} \approx I_{E2} = 3\text{ mA}$;

$V_o = -10\text{ V} + 3\text{ mA}\cdot 2\text{ k}\Omega = -4\text{ V}$.

Circuits with bipolar transistors

10.6. $r_e = 25/I_E\ \Omega$, with I_E the emitter current in mA.

a. $V_B = V^- + \dfrac{R_2}{R_1 + R_2}(V^+ - V^-) = -20 + \dfrac{40}{11} = -16.36\text{ V}$;

$V_E = V_B - 0.6 = -16.96\text{ V}$;

$I_E = \dfrac{V_E - V^-}{R_E} = \dfrac{-16.96 + 20}{5600} = 0.65\text{ mA} \rightarrow r_e = \dfrac{25}{0.65} = 38.5\ \Omega$.

b. $V_B = V^+\dfrac{R_2}{R_1 + R_2} = 7.5\text{ V}$; $V_E = 6.9\text{ V}$;

$I_E = \dfrac{V_E}{R_E}\ 6.9\text{ mA} \rightarrow r_e = 3.6\ \Omega$.

c. For T_1:

$V_B = \dfrac{R_2}{R_1 + R_2}V^+ + \dfrac{47}{147}\cdot 7.5 = 2.4\text{ V}$; $V_E = 1.8\text{ V}$;

$I_E = \dfrac{V_E}{R_4} = 0.1\text{ mA} \rightarrow r_{e1} = 250\ \Omega$.

For T_2: $V_B = V^+ - I_{C1}R_3 = 7.5 - 0.1\cdot 22 = 5.3\text{ V}$;

$V_E = V_B + 0.6 = 5.9\text{ V}$;

$I_E = \dfrac{V^+ - V_B}{R_5} = \dfrac{1.6}{1500} \approx 1\text{ mA} \rightarrow r_{c2} \approx 25\ \Omega$.

10.7. a. CE-stage: $A = -\dfrac{R_C}{R_E} = -\dfrac{33\text{k}}{5.6\text{k}} = -5.9$.

b. Emitter follower: $A \approx 1$.

c. This is a two-stage voltage amplifier with decoupled emitter resistances. As $\beta \rightarrow \infty$, the base current through T_2 is zero; the first stage is not loaded by the second stage. In that case: $A = A_1 \cdot A_2$.

$A_1 = -\dfrac{R_3}{r_{e1}} = -88$; $A_2 = -\dfrac{R_6}{r_{e2}} = -108 \rightarrow A = +9504$.

10.8. The transfer of an emitter follower with $\beta \rightarrow \infty$ is $\dfrac{R_E}{r_e + R_E} \approx 1 - \dfrac{r_e}{R_E}$.

The relative deviation from 1 is thus $-\dfrac{r_e}{R_{E=}} = -\dfrac{3.6}{1000} = -0.36\%$.

10.9. The transfer of an emitter follower with $\beta >> 1$ is

$$\frac{\beta R_E}{r_b + \beta(r_e + R_E)} \approx 1 - \frac{r_e}{R_E} - \frac{r_b}{\beta R_E}$$

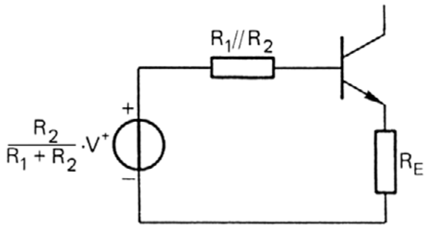
The relative deviation from 1 is thus $-\dfrac{r_e}{R_E} - \dfrac{r_b}{\beta R_E} = -0.46\%$.

10.10. The input resistance is R_1 // R_2 // $[r_b + (1 + \beta)(r_c + R_E)] \approx R_1$ // R_2 // βR_E = 100 kΩ // 100 kΩ // 100 kΩ = (100/3) kΩ.

10.11. In Exercise 10.7, the transfer is $A = A_1 \cdot A_2$, with $A_1 = -(R_3/r_{e1})$.
The input resistance r_{i2} of the second stage thus becomes parallel to R_3; so replace R_3 in the formulas with R_3 // r_{i2}. The input resistance is approximately $r_{i2} = \beta r_{e2}$ = 2500 Ω, so R_3 // $r_{i2} \approx$ 2240 Ω. This is almost a factor of 10 smaller than the value previously calculated.

10.12. First, find V_B, then I_E. As β is not infinite, the base current cannot be ignored. A Thévenin equivalent of the input circuit is depicted below (see Figure 5.2a).

$$V_B = \frac{R_2}{R_1 + R_2} V^+ - (R_1 \,//\, R_2) I_B$$



Obviously, the base current affects the transistor bias. If the second term is small, it can be ignored; not however in this example:

$$V_B = \frac{R_2}{R_1 + R_2} V^+ - (R_1 \,//\, R_2) \frac{I_E}{\beta} = V_E + 0.6;\ V_E = I_E R_E;$$

$$I_E \left(R_E + \frac{R_1 // R_2}{\beta} \right) = \frac{R_2}{R_1 + R_2} V^+ - 0.6 \rightarrow I_E = \frac{7.5 - 0.6}{1000 + 500} = 4.6 \text{ mA},\ r_e = 5.43\ \Omega.$$

Usually the designer ensures that with R_1 and R_2 the bias is independent of the base current.

11. Field-effect transistors

Properties of field-effect transistors

11.1. The gate-source voltage at $V_{DS} = 0$, for which the channel is non-conducting (see Section 11.1.1).

11.2. In this region there are (virtually) no free charge carriers (see Section 9.1.1).

11.3. The gate current of a JFET consists of the reverse currents of the reverse biased junctions (gate-source and gate-drain). In a MOSFET, the gate is completely isolated from the rest of the device by an oxide layer, so I_G is almost zero (smaller than 10^{-12} A).

11.4. $V_G = 0$; $V_{GS} = -4$ V $\rightarrow V_S = 4$ V;

$$I_S = \frac{V_S}{R_S} = \frac{4}{1500};\ I_D = I_S\ (\text{as } I_G = 0);$$

$$V_D = V^+ - I_D R_D = 20 - \frac{4 \cdot 3300}{1500} = 11.2 \text{ V}.$$

11.5. The maximum resistance of the network (10 kΩ) is obtained for $R_{FET} \rightarrow \infty$ so for $V_{GS} < V_P$: $R = R_1$ = 10 kΩ.
The lowest resistance (1 kΩ) is obtained for $V_{GS} = 0$, R_{FET} = 800 Ω:
$R = R_1$ // $(R_2 + R_{FET})$ = 1 kΩ $\rightarrow R_2$ = 311 Ω.

11.6. To avoid that pn-junctions at the source and drain contacts may affect the signal behaviour of the transistor (see Section 11.1.2).

11.7. The names refer to the type of charge carriers that are responsible for the conduction in the material. In a bipolar transistor these are both holes and electrons; in an n-channel FET only electrons and in a p-channel FET only holes.

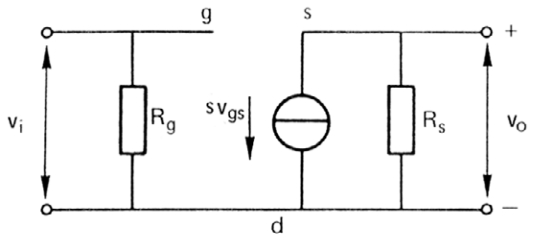
Circuits with field-effect transistors

11.8. $I_D = \frac{5}{3} V_{GS} + 5$ (I_D in mA).

11.9. $V_G = 0$, thus $V_{GS} = -V_S$; $I_D = I_S = V_S/R_S$;
substitution in the equation of Exercise 11.8 results in: $I_D = \frac{5}{6}$ mA.

11.10. Similar to Exercise 11.9: $I_D = 2.5$ mA; $V_D = V^+ - I_D R_D = 10 - 2.5 \cdot 1.8 = 5.5$ V.

11.11.



The power supply voltage has no effect on the calculation of the signal properties.

- voltage transfer (at $i_o = 0$):

$$v_o = g v_{gs} R_S = g(v_k - v_s) R_S = g(v_i - v_o) R_S \rightarrow \frac{v_o}{v_i} = \frac{g R_S}{1 + g R_S} = \frac{4}{1+4} = 0.8;$$

- input resistance (at $i_o = 0$): $\frac{v_i}{i_i} = r_i = R_G = 1$ MΩ;.

- output resistance (at $v_i = 0$):

$$v_o = (i_o + g v_{gs}) R_S = (i_o - g v_o) R_S \rightarrow \frac{v_o}{v_i} = r_o = \frac{R_S}{1 + g R_S} = 400\,\Omega.$$

11.12. a. T_1 is connected as a source follower. The total source resistance is R in series with the output resistance of T_2 (at the drain). The latter is infinite (current source character) so the voltage transfer of the source follower is 1: the AC voltages at the source of T_1 and the drain of T_2 are both equal to v_i. The DC (average) value of v_o is $V_{GS}(T_1) + I_D R$ lower than that of v_i.

b. $V_{GS}(T_2) = 0 \rightarrow I_D\,(T_2) = 2$ mA;

$I_D\,(T_1) = I_D\,(T_2) = 2$ mA $\rightarrow V_{GS}\,(T_1) = 0$.

The required voltage drop is 3 V, which is equal to the voltage drop across R: $I_D R = 3$, so $R = 1500$ Ω.

c. The maximum peak voltage depends on the V_{DS}-I_D characteristic of the FET (Figure 11.2c); but in any case, all pn-junctions must remain reverse biased. From the characteristic it follows:

for T_1: $V_{DS} > 0$; $V_{Smax} = V_{Gmax} = 10$ V $\rightarrow \hat{v}_{max} < 3$V;

for T_2: $V_{Dmin}(T_2) = V_{Smin}(T_1) - 3 = 0$ V $\rightarrow \hat{v}_{max} < 4$V;

$V_{GS}(T_1)$ and $V_{GS}(T_2)$ are 0 V. $V_{DG}(T_1) = 3$ V and $V_{DG}(T_2) = 4$ V, hence all pn-junctions are reverse biased. The condition for T_2 is always fulfilled if (for T_1) $\hat{v}_{max} < 3$V.

From Figure 11.2c it appears that with this large value of the input voltage T_1 operates in the pinch-off region and is no longer linear. For this reason, the input voltage should be kept lower than the value calculated before.

12. Operational amplifiers

Amplifier circuits with ideal operational amplifiers

12.1. A virtual ground is a point that is at ground potential without being connected to ground. At proper feedback, the voltage between the two inputs of an operational amplifier is zero. At grounded positive terminal the negative terminal is virtually grounded. In a non-inverting configuration, none of the input terminals is grounded.

12.2. a. Superposition of the separate contributions of V_1 and V_2 to the output voltage:

$$V_o = -\frac{R_3}{R_1}V_1 - \frac{R_3}{R_2}V_2 = -15{\cdot}2 - 12.2{\cdot}(-3) = +6.6 \text{ V}.$$

b. In the ideal case (equal resistances) $V_o = 0$. In the general case,

$$V_o = \frac{R_1R_4 - R_2R_3}{R_1(R_3 + R_4)}V_i \text{ (see Figure 12.5);}$$

with $R_{1,2,3,4} = R(1 \pm \varepsilon)$, the maximum output voltage is

$$V_o \approx \frac{4\varepsilon}{2}V_i = 2 \cdot 0.3 \cdot 10^{-2} \cdot 10 = 60 \text{ mV}$$

c. $V_o = -\infty$ (in a real situation: limited by the negative power supply voltage).

d. Superposition of the separate contributions of V_i and I_i to the output:

$$V_o = -\frac{R_2}{R_1}V_i + R_2I_i = -1 \cdot 0.6 + 15 \cdot 10^3 \cdot 40 \cdot 10^{-6} = 0.$$

e. The current through the negative input is zero, so the voltage across R_3 is zero too. The voltage at the negative input is V_o; the circuit behaves as a buffer amplifier. As the output impedance is zero, the load resistor R_4 does not affect the transfer.

$$V_o = \frac{R_2}{R_1 + R_2}V_i = \frac{120}{300} \cdot 5 = 2 \text{ V}.$$

f. $V_3 = -(R_t/R_1)V_1$ (inverting amplifier); $V_o = V_5$ (buffer amplifier);

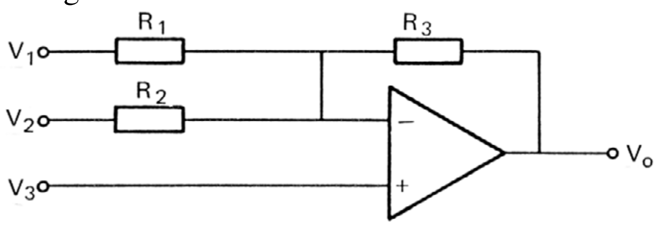
$$V_5 = \frac{R_3}{R_2 + R_3}V_3 + \frac{R_2}{R_2 + R_3}V_4 \text{ (superposition).}$$

Substitution of the numerical values:

$$V_3 = -\frac{18}{0.6} \cdot 0.3 = -9 \text{ V}; \; V_o = \frac{18}{40} \cdot (-9) + \frac{22}{40} \cdot 1 = -4.05 + 0.55 = -3.5 \text{ V}.$$

12.3. $R_i = R_1 > 5$ kΩ; take $R_1 = 5.6$ kΩ (see Section 7.1.1). $R_2/R_1 = 50$, so $R_2 = 280$ kΩ (for instance two resistances of 560 kΩ in parallel).

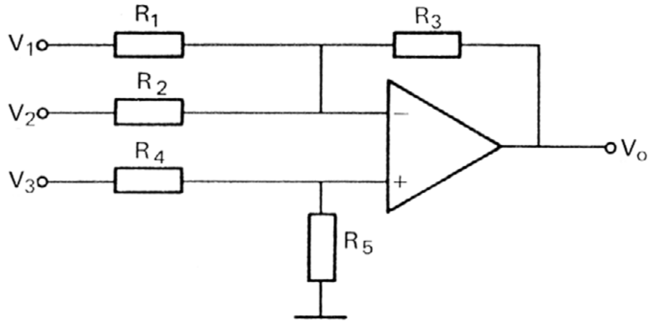
12.4. The circuit must have two inverting and one non-inverting input. Examine the next configuration.



In this circuit:

$$V_o = -\frac{R_3}{R_1}V_1 - \frac{R_3}{R_2}V_2 + \left(1 + \frac{R_3}{R_1 /\!/ R_2}\right)V_3,$$ with the conditions: $R_3/R_1 = 10$, $R_3/R_2 = 5$ and $R_3 = R_1/\!/R_2$.

There is no solution: the V_o/V_3 gain is always larger than the other two gain factors V_o/V_1 and V_o/V_2. The voltage V_3 should first be attenuated, for instance with the voltage divider with resistances R_4 and R_5.



Now: $V_o = -\frac{R_3}{R_1}V_1 - \frac{R_3}{R_2}V_2 + \left(1 + \frac{R_3}{R_1//R_2}\right)\left(\frac{R_5}{R_4 + R_5}\right)V_3.$

We have several options. Take, for instance, R_3 =100 kΩ; then R_1 = 10 kΩ and R_2 = 20 kΩ. Furthermore: $R_1//R_2$ = 20/3 kΩ, so 1 + $R_3/(R_1//R_2)$ = 16. The attenuation of the voltage divider must be 8, and that can be achieved by (for instance) choosing R_3 = 10 kΩ, R_4 = 70 kΩ.

12.5. Let the two resistances be R_{b1} and R_{b2}. The common-mode transfer remains 1: the voltage drop across R_a is still zero, hence the voltage drop across R_{b1} and R_{b2} is zero too. If the differential amplifier is perfect then, the common-mode transfer will be zero.

The differential mode transfer is found with:

$V_1' = V_1 + \frac{R_{b1}}{R_a}V_d$ and $V_2' = V_2 - \frac{R_{b2}}{R_a}V_d$, from which follows:

$$V_1' - V_2' = V_1 - V_2 + \frac{R_{b1} + R_{b2}}{R_a}V_d = \left(1 + \frac{R_{b1} + R_{b2}}{R_a}\right)V_d,$$

Conclusion: unequal values for R_{b1} and R_{b2} do not affect the CMRR; the resistance values determine the transfer for differential voltages.

Non-ideal operational amplifiers

12.6. Put a voltage source V_{off} in series with the non-inverting input of the operational amplifier. Then:

$$V_o = \left(1 + \frac{R_2}{R_1}\right)V_{off} = \left(1 + \frac{10}{2.2}\right)\cdot 0.4 = 2.22\,\text{mV}.$$

12.7. Without R_3 the output voltage will be $V_o = R_2 I_{bias} = 10^4 \cdot 10^{-8} = 0.1$ mV.
$R_3 = R_1//R_2 = 10 \cdot 2.2/12.2 = 1.8$ kΩ. So $V_o = I_{off}R_2 = 0.01$ mV.

12.8. As the inverting input-terminal is virtually grounded, $R_i = V_i/I_i = R_1$ = 2200 Ω. The amplifier has ideal properties, so $R_o = 0$.

12.9. At unity feedback, the gain is 1; the bandwidth f_t = 1.5 MHz. The gain is A=10/2.2, so the bandwidth is equal to $1.5 \cdot 10^6 \cdot 2.2/10$ = 330 kHz.

12.10. The input resistance has the value of R_1. Take R_1 = 12 kΩ. The required gain is $-R_2/R_1$ = –30, so R_2 = 360 kΩ. The voltage on the slider of R_3 may vary from –15 to +15 V; the compensation voltage at the output should vary from +1.2 to –1.2 V, so the transfer (for the compensation voltage) must be –15/1.2 = –12.5. As the resistance of the potentiometer depends on the position of the slider, so too does the transfer. For the slider at the extreme ends of the potentiometer, the transfer is $-R_2/R_4$, so R_4 = 12.5 · 12 = 150 kΩ. Take for instance a potentiometer of 100 kΩ; the value is not critical, it mainly determines the current that must be supplied by the power source.

13. Frequency-selective transfer functions with operational amplifiers

Circuits for time-domain operations

13.1. $v_o = \frac{1}{C}\int_0^{10}(V_{off}/R + I_{bias})dt + V_{off} \approx 10^6(10^{-4}/10^4 + 10^{-8})\cdot 10 = 0.2\text{V}.$

13.2. If both bias currents are equal, the contribution of I_{bias} to the output will be eliminated, hence:

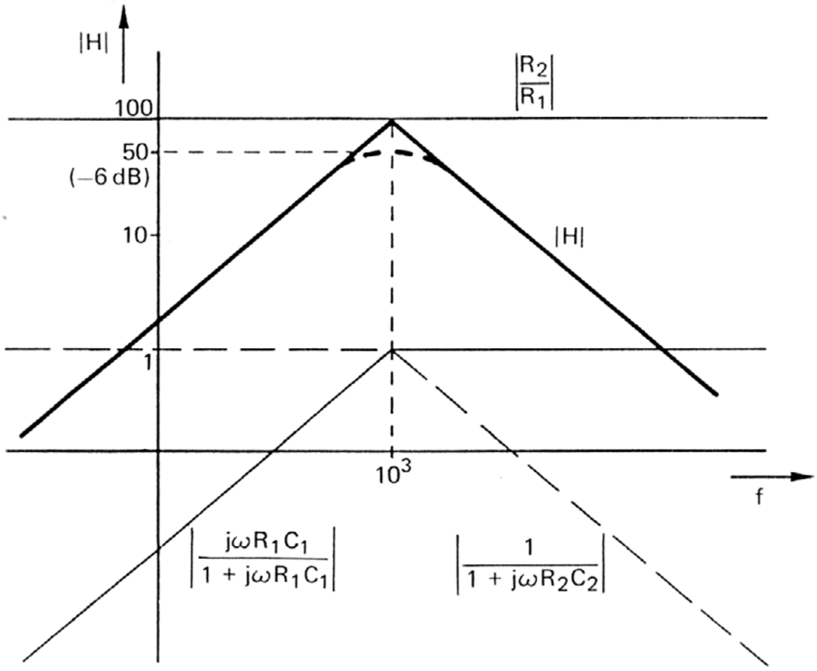
$$v_o = \frac{1}{C}\int_0^{10}(V_{off}/R)dt = 0.1\text{V}.$$

13.3. The output voltage reaches the end value $v_o = (1 + R_o/R)V_{off}$ = 10.1 mV.

13.3. a. The complex transfer function is:

$$H = -\frac{Z_2}{Z_1} = -\frac{R_2}{R_1}\cdot\frac{j\omega R_1C_1}{1+j\omega R_1C_1}\cdot\frac{1}{1+j\omega R_2C_2}.$$

The amplitude characteristic is the sum of the next three curves: R_2/R_1 (frequency-independent transfer 100); $j\omega R_1C_1/(1 + j\omega R_1C_1)$ (a first-order high-pass characteristic with break point at $f = 1/2\pi R_1C_1 \approx 10^3$) and $1/(1 + j\omega R_2C_2)$ (first-order low-pass characteristic with break point at $f = 1/2\pi R_2C_2 = 10^3$).



(The deviation from the asymptotic approximation at the break point is –3dB for the last two curves, so it is –6 dB (or a factor of 2) for the total transfer, see dashed curve).

b. The phase shift follows from the argument of the transfer function (we disregard the inversion minus sign):

$$\arg H = \frac{\pi}{2} - \tan^{-1}\omega R_1C_1 - \tan^{-1}\omega R_2C_2 = \frac{\pi}{2} - 2\tan^{-1}(10^{-3}f).$$

The deviation from 90° is 2arctan($10^{-3} f$); this is more than 10° for $f \geq 87$ Hz.

13.5. The input resistance has value R_1, so take R_1 = 10 kΩ. The gain in the P-region (the region of high frequency, where C can be viewed as a short-circuit) is $-R_2/R_1$, so R_2 = 20 kΩ.
The break point between the I- and P-region is at $\omega_k = 1/R_2C$. As $f_k > 100$ Hz, $1/2\pi R_2C > 100$ or $C < 80$ nF. Since near the break point the deviation from the integrating character is rather great, we will take for instance C = 10 nF.

Circuits with high frequency-selectivity

13.6. Rewrite the transfer function as

$$H = \frac{a_0 + a_1 j\omega + a_2(j\omega)^2}{1 + j\omega / Q\omega_0 - \omega^2 / \omega_0^2}.$$

$$H = \frac{1}{3}\frac{1+2j\omega\tau - \omega^2\tau^2}{1 + j\omega\tau/12 - \omega^2\tau^2},$$ from which follows:

$\omega_0 = 1/\tau = 10^3$ rad/s

$\omega\tau/12 = \omega/Q\omega_0 = \omega\tau/Q$ so Q=12;

$$|H(\omega \to \infty)| = \frac{1}{3};\ |H(\omega \to 0)| = \frac{1}{3};\ |H(\omega_0)| = \frac{1}{3}\cdot\frac{2j\omega\tau}{j\omega\tau/12} = 8..$$

13.7. $$H = \frac{Z_2}{Z_1 + Z_2} = \frac{j\omega R_2C_1}{1 + j\omega(R_2C_1 + R_1C_1 + R_2C_2) - \omega^2 R_1R_2C_1C_2} = \frac{j\omega\tau}{1 + 3j\omega\tau - \omega^2\tau^2}.$$

Compared with the general expression this results in:

$\omega_o = 1/\tau = 100$ rad/s or $f_0 = 100/2\pi \approx 15.9$ Hz.

$3\omega\tau = \omega/Q\omega_0 = \omega\tau/Q$, or $Q = 1/3$.

13.8. Replace all the resistors with capacitors and vice versa. The new transfer function then becomes:

$$H = \frac{-\omega^2 R_1R_2C_1C_2}{1 + j\omega R_1(C_1 + C_2) - \omega^2 R_1R_2C_1C_2}.$$

$$|H|^2 = \frac{(\omega^2 R_1R_2C_1C_2)^2}{(1 - \omega^2 R_1R_2C_1C_2)^2 + \omega^2 R_1^2(C_1 + C_2)^2}.$$

The condition for a Butterworth filter is:

$-2R_1R_2C_1C_2 + R_1^2(C_1 + C_2)^2 = 0$, or $R_2/R_1 = (C_1 + C_2)^2/2C_1C_2$..

Take $C_1 = C_2 = C$, hence $R_2 = 2R_1$.

13.9. The output voltage of the differential amplifier is denoted as V_x, that of the upper integrator V_y, so that:

$$V_o = -\frac{1}{j\omega R_1C_1}V_x;\ V_y = -\frac{1}{j\omega R_2C_2}V_o;$$

$$V_x = -\frac{R_4}{R_3}V_y + \left(1 + \frac{R_4}{R_3}\right)\frac{R_6}{R_5 + R_6}V_i + \left(1 + \frac{R_4}{R_3}\right)\frac{R_5}{R_5 + R_6}V_o,$$

Elimination of V_x and V_y results in:

$$\frac{V_o}{V_i} = -\frac{(1+R_3/R_4)R_6}{R_5+R_6} \cdot j\omega R_2 C_2$$
$$\times \left(1 + j\omega R_2 C_2 \frac{(1+R_3/R_4)\cdot R_5}{R_5+R_6} - \omega^2 R_1 R_2 C_1 C_2 R_3/R_4\right)^{-1}$$

so $\omega_0 = 1\sqrt{R_1R_2C_1C_2R_3/R_4}$ and $Q = \frac{R_5+R_6}{(1+R_3/R_4)R_5}\sqrt{R_1C_1R_3/R_2C_2R_4}.$

Q can be adjusted independently of ω_0 by R_5 and/or R_6. To vary ω_0 independently of Q, the condition $R_1C_1 = R_2C_2$ must be fulfilled. If further, for the sake of simplicity, $R_1 = R_2 = R$, $C_1 = C_2 = C$ and $R_3 = R_4$, then $\omega_0 = 1/RC$ and $Q = (R_5 + R_6)/2R_5$.

14. Non-linear signal processing with operational amplifiers

Non-linear transfer functions

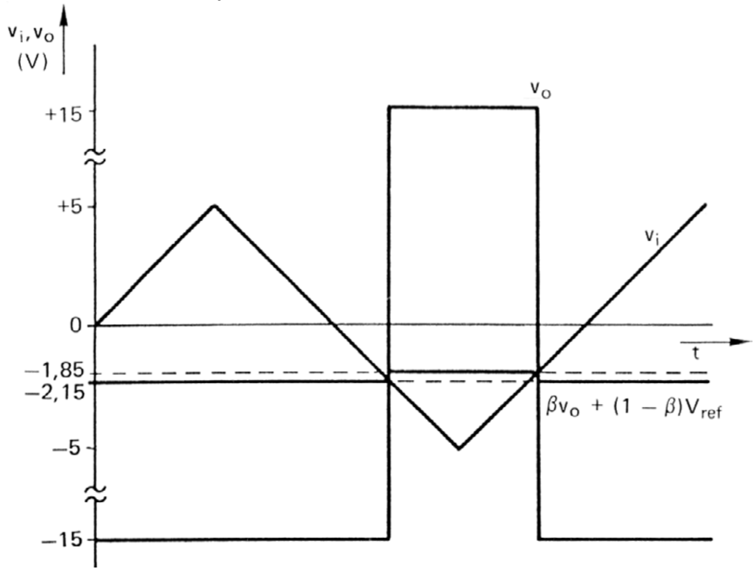
14.1. Error due to inaccuracy in V_R: ±2 mV;
error due to loading: about $-R_b/(R_i + R_b) = -1/17 \approx -6\%$;
absolute error due to voltage offset: ±2 mV ± (70·8 μV) = ±2.56 mV.
absolute error due to input current: $\pm I_b R_b = \pm 5$ mV.

14.2. The switching levels are determined by the two possible voltages at the plus-terminal of the operational amplifier; these are:

$$\frac{R_2}{R_1+R_2}V \pm \frac{R_1}{R_1+R_2}E \approx V \pm \frac{R_1}{R_2}E = -2 \pm 0.01 \cdot 15.$$

E is the absolute value of the power supply voltage. The switching levels are –1.85 and –2.15 V; the hysteresis is 0.3 V.

14.3.



14.4. The break point in the transfer characteristic occurs at value v_i where the current through D_1 (or D_2) is zero. This current equals:

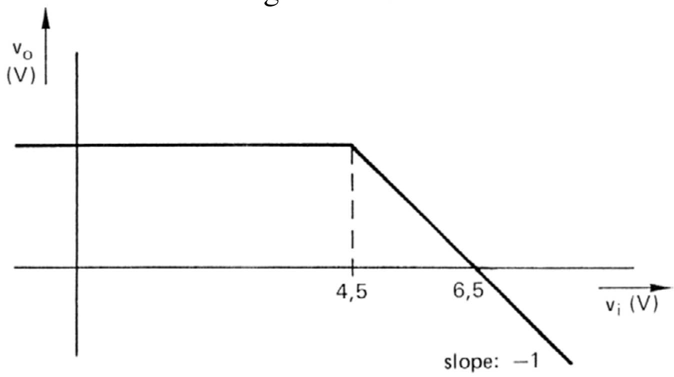
$$\frac{V_{ref,1} - V_{ref,2}}{R_3} + \frac{v_i - V_{ref,2}}{R_1},$$

so the break point occurs at $v_i = V_{ref,2} = -\frac{R_1}{R_3}(V_{ref,1} - V_{ref,2})$

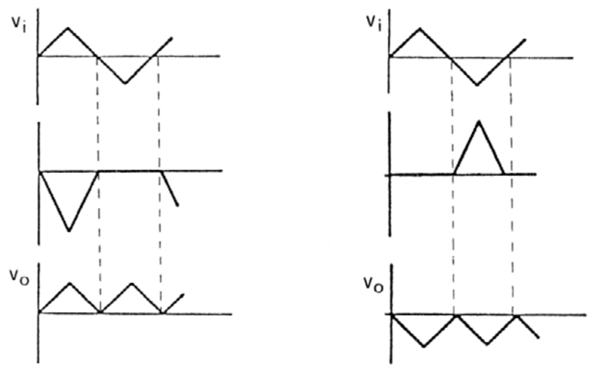
The corresponding output voltage is just equal to $V_{ref,2}$. The output voltage in the linear region (D_2 is forward biased) amounts to

$$-\frac{R_2}{R_3}V_{ref,1} - -\frac{R_2}{R_1}v_i + \left(1 + -\frac{R_2}{R_1 // R_3}\right)V_{ref,2}$$

The characteristic is given below.



14.5.



Left: signals in the circuit of Figure 14.8. Right: signals in the case of reversed diodes.

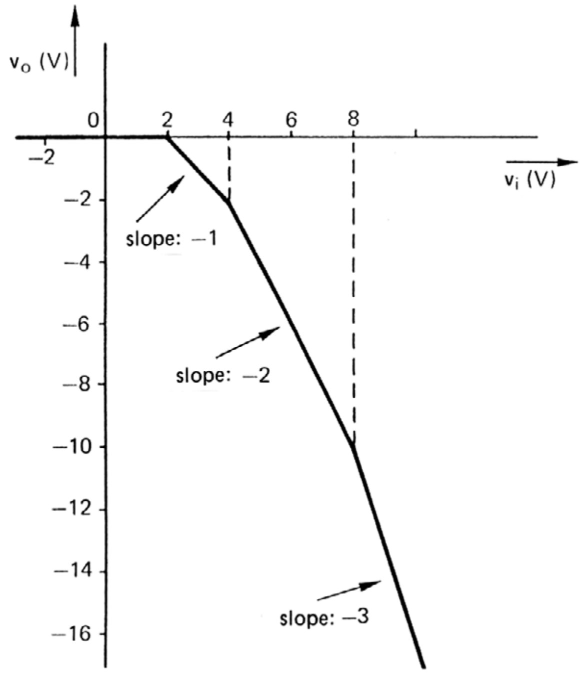
14.5. The break points occur for those values of v_i where the diodes just become forward biased; the conditions are:

$$\frac{770}{770+2310}V_i = 0.5\,\text{V};\ \frac{330}{330+2310}V_i = 0.5\,\text{V and}$$

$$\frac{154}{154+2310}V_i = 0.5\ \text{V, or: 2 V, 4 V and 8 V.}$$

The slope for each separate section equals –1, because they amount to $-R_7/R_1$, $-R_7/R_3$ and $-R_7/R_5$, respectively.

The output voltage at the break points is: 0 V; 0 + (–1)·2 = –2 V and –2 + (–2)·4 = –10 V. The transfer looks like this:



Non-linear arithmetic operations

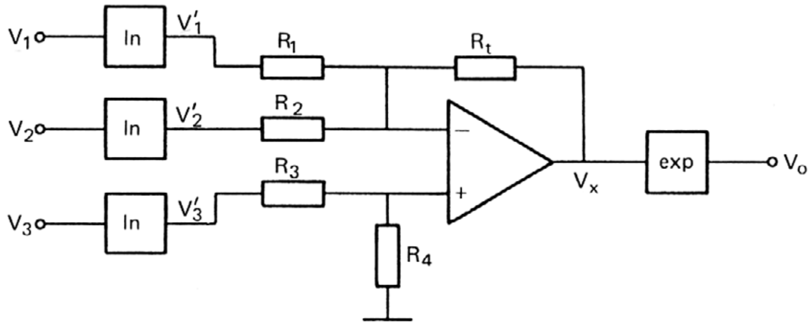
14.7. $v_{o2} = K_E{\cdot}\exp(v_{i2}/V_E) = K_E{\cdot}\exp(v_{o1}/V_E) = K_E{\cdot}\exp[K_L\ln(v_{i1}/V_L)/V_E] = K_E{\cdot}\exp[\ln(v_{i1}/V_L)^{K_L/V_E}]$; $v_{o2} = v_{i1}$ if $K_L = V_E$ and $K_E = V_L$.

14.8. a. $v_o = V_x = v_y$; $v_i = Kv_xv_y$, hence $v_i = Kv_o^2$ or $v_o = \sqrt{v_i/K} = \sqrt{v_i}$.

b. Both input voltages of the operational amplifier are zero, so: $-v_i/R_1 = (v_o/R_3) + Kv_o^2/R_2$.

14.9. $v_o = -10^{v_q/10} = -10^{-Av_p/10} = -10^{A\log(v_i/10)/10} = -(v_i/10)^{A/10} = -\sqrt{v_i/10}$

14.10. The weighted addition of ln v_1 and ln v_2, the subtraction of ln v_3, followed by exponential conversion:



$$V_x = -\frac{R_t}{R_1}v_1' - \frac{R_t}{R_2}v_2' + \left(1 + \frac{R_t}{R_1//R_2}\right)\frac{R_4}{R_3 + R_4}v_3' \text{ with } v_{1,2,3}' = \ln(v_{1,2,3})$$

The resistance values must satisfy $R_t/R_1 = 1/3$ and $R_t/R_2 = 2/3$, from which follows: $R_t/(R_1//R_2) = 1$, hence $R_4/(R_3 + R_4) = 1/2$, or $R_3 = R_4$.

14.11. If the bias currents are zero, then $v_o = \log 1 = 0$. With non-zero bias currents:

$$\log\frac{10^{-5} \pm 10^{-9}}{10^{-4} \pm 10^{-9}} \pm \log 0.1 + \log(1 \pm 10^{-4}) + \log(1 \pm 10^{-5});$$

For $I_1 = 10$ μA and $I_2 = 100$ μA the output voltage at zero bias currents is log 0.1 = –1 V. With bias currents this is:

$$\log\frac{10^{-5}\pm 10^{-9}}{10^{-4}\pm 10^{-9}}\approx \log 0.1+\log(1\pm 10^{-4})+\log(1\pm 10^{-5});$$

the error is thus about ±47.8 μV.

15. Electronic switching circuits

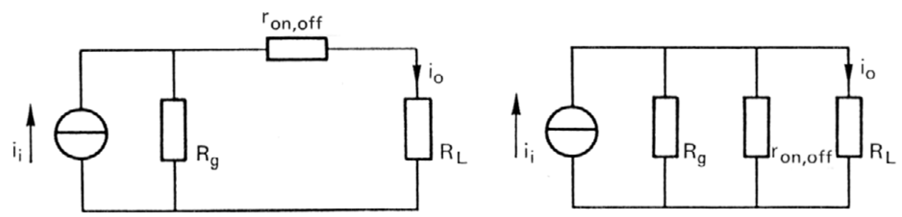
Electronic switches

15.1. In the on-state, $v_o/v_i = R_L/(R_L + R_g + r_{on}) \approx 1 - R_g/R_L - r_{on}/R_L$.
R_g/R_L= 0.02%, so the error due to r_{on} must be less than 0.08%. Hence, $r_{on}/R_L < 8{\cdot}10^{-4}$ thus $r_{on} < 40\ \Omega$.
In the off-state, $v_o/v_i = R_L/(R_L + R_g + r_{off}) \approx R_L/r_{off}$; this must be less than 10^{-3}, so $r_{off} >$ 50 MΩ.

15.2. In the on-state (switch off), $v_o/v_i = R_L/(R_gR_L/r_{off} + R_g + R_L) \approx 1 - R_g/r_{off} - R_g/R_L$; the error due to the switch must be less than 0.08%, so $r_{off} > 12.5$ kΩ.
In the off-state (switch on), $v_o/v_i = r_{on}/(R_gr_{on}/R_L + r_{on} + R_g) \approx 1 - r_{on}/R_g$; this must be less than 10^{-3}, hence $r_{on} < 0.01\ \Omega$.

15.3. In the on-state, $v_o/v_i \approx 1 - R_L/r_{off} - r_{on}/ r_{off} - r_{on}/R_L - R_g/R_L$. R_g/R_L contributes 0.02% to the error, so 0.08% remains for the error terms R_g/r_{off}, r_{off}/r_{on} and r_{on}/R_L. As $r_{off} >> r_{on}$, the condition is $R_g/r_{off} + r_{on}/R_L < 0.08\%$. Equal partitioning of these errors over r_{on} and r_{off} results in $R_g/r_{off} < 4{\cdot}10^{-4}$ and $r_{on}/R_L < 4{\cdot}10^{-4}$, so $r_{off} > 25$ kΩ and $r_{on} < 20\ \Omega$.
In the off-state, $v_o/v_i \approx r_{on}/r_{off}$, from which follows: $r_{off} > 10^3 r_{on}$; this requirement is fulfilled with the conditions found before.

15.4. Equivalent circuits for the series and shunt switches:



$i_o / i_i = R_g / \left(R_g + R_L + r_{on/off}\right)$; $i_o / i_i = \left(R_g // r_{on/off}\right) / \left(R_g // r_{on/off} + R_L\right)$;

For the series-shunt configuration:

$$i_o / i_i = \left(1+\frac{r_{s1}}{R_g}+\frac{R_L}{r_{s2}}+\frac{r_{s1}R_L}{r_{s2}R_g}+\frac{R_L}{R_g}\right)^{-1},$$

with $r_{s1} = r_{on}$, $r_{s2} = r_{off}$ or vice versa.
Assuming $R_g >> R_L$ and $r_{off} >> r_{on}$, the next table can be derived:

	r_{on}	r_{off}
series	$<< R_g$	$>> R_g$
shunt	$<< R_L$	$>> R_L$
series/shunt	$<< R_g$	$>> R_L$

15.5. $r_{on} = 2r_d//2r_d = r_d$. At 1 mA, $r_d = 25\ \Omega$, so at 5 mA this is 5 Ω.

15.6. When the first circuit is the on-state, r_{on} is in series with the source resistance and V_{off} is in series with the offset voltage of the operational amplifier. The switch increases the offset (V_{off} + $I_{bias}r_{on}$) of the system and it increases the load error (relative load error $-r_{on}/R_i$). In the second circuit, the offset V_{off} of the switch is reduced to V_{off}/A at the system output point, and the operational amplifier's gain is A.. The on-resistance of the switch only contributes with r_{on}/A to the output resistance of the circuit. In view of these parameters, it is the second circuit that is preferred to the first one.

15.7. For $i_s = 0$, $v_{gs} = 0$ (no current through R) so the FET conducts, irrespective of the resistance R.
For $i_s = 2$ mA, $v_{gs} = -i_sR$; this must be less than –6 V, so $R > 3$ kΩ.

15.8. To keep the circuit in the on-state, the current through R must be zero irrespective of v_i. This is achieved if the diode is reverse biased, so $v_s > v_{i,max} - V_d$, or $v_s > 3 - 0.6 = 2.4$ V.
In the off-state, for each value of v_i, v_{gs} must be less than the lowest pinch-off voltage, hence $v_{gs} < -6$ V. This occurs when the diode is forward biased, so $v_s < v_{i,min} + v_{gs} - V_d$, or $v_s < -3 - 6 - 0.6 = -9.6$ V.

Circuits with electronic schwitches

15.9. In the on-state, r_{on1} is in series with R_1 and r_{on2} is in series with R_2. Furthermore, $r_{on1} = r_{on} \pm \Delta r_{on1}$, $r_{on2} = r_{on} \pm \Delta r_{on2}$ and $\Delta r << r_{on}$.
The differential gain is $A_v \approx R/(R + r_{on1})$
The common-mode gain is

$$A_c = \frac{v_{oc}}{v_{ic}} = -\frac{R_3}{R_1 + r_{on1}} + \frac{R_4}{R_2 + R_4 + r_{on2}}\left(1 + \frac{R_3}{R_1 + r_{on1}}\right)$$

$$= \frac{R}{R + r_{on1}}\left(\frac{r_{on1} - r_{on2}}{2R + r_{on2}}\right) \approx \frac{R}{R + r_{on1}} \cdot \frac{\pm\Delta r_{on1} \pm \Delta r_{on2}}{2R + r_{on2}}.$$

The CMRR is $\left|\frac{A_v}{A_g}\right| = \left|\frac{2R + r_{on2}}{\pm 2\Delta r}\right| = \pm 5050.$

15.10. In the track mode the output voltage at $v_i = 0$ equals
$v_o = |V_{off}| + |I_{bias}(r_{on} + R_g)| = 250\ \mu$V.
The value of C_H is not relevant here.

15.11. $dv_o/dt = I/C_H = 1$ V/s. $T = 1/f = 10$ ms.
After 1 period (10 ms) $v_o = 8$ V + 10 mV; after 100 periods (1 s) $v_o = 8 + 1 = 9$ V.

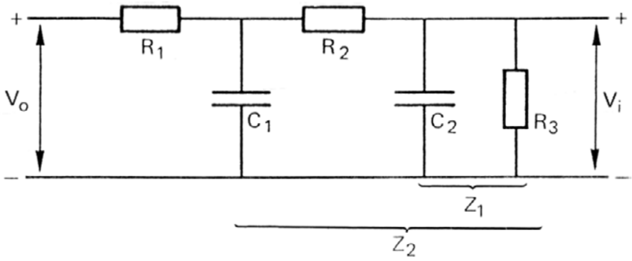
16. Signal generation

Sine wave oscillators

16.1. There are three reasons.
a. To start the oscillation: first, α must be negative to let the amplitude increase up to the required value; then, α should return to zero.
b. To determine the amplitude: the solution of the differential equation does not depend on the amplitude; it can only be varied by controlling the damping factor α.
c. To stabilize the amplitude; factor α is subject to drift, so the amplitude will change even when α is very close to zero.

16.2. Each electronic system generates noise; the noise with frequencies close to the oscillation frequency will be amplified for $\alpha < 0$, so the oscillation starts on the system noise.

16.3. a. The transfer function of two low-pass RC networks in series and the latter be loaded with R_3 (the input resistance of the integrator) can be deduced from the next figure, where v_i is the input voltage of the integrator. The transfer is found by applying two times the voltage divider rule (see also Exercise 3.3).



$$H_1 = \frac{Z_1}{R_2 + Z_1} = \frac{1}{2 + j\omega\tau}(\tau = RC)$$

$$H_2 = \frac{Z_2}{R_1 + Z_2} = \frac{2 + j\omega\tau}{3 + 4j\omega\tau - \omega^2\tau^2}$$

$$H = H_1 H_2 = \frac{1}{3 + 4j\omega\tau - \omega^2\tau^2}$$

The transfer of the integrator is $-1/j\omega RC_t$. The product of both transfers must be 1, hence: $1/(3 + 4j\omega\tau - \omega^2\tau^2)\cdot(-1/j\omega RC_t) = 1$. The imaginary part of the left-hand side of this equation is zero, which results in $\omega^2\tau^2 = 3$ or $f = \sqrt{3}/2\pi\tau$. The real part is 1, so $4\omega^2\tau RC_t = 1$. This results, together with the expression of $\omega\tau$ found before, in the condition $C_t = C/12$.

b. The voltage at the inverting input terminal of the operational amplifier is $\beta_1 v_o$, with $\beta_1 = R_1/(R_1 + R_2)$; the voltage at the non-inverting input is $\beta_2(\omega)v_o$, with $\beta_2(\omega) = .j\omega\tau/(1 + 3j\omega\tau - \omega^2\tau^2)$ (see figure 8.10a). Both voltages are equal, so $\beta_1 = \beta_2(\omega)$. As β_1 is real, the imaginary part of β_2 must be zero, hence $\omega^2\tau^2 = 1$. So $\beta_2 = 1/3$, which is also the value of β_1. Conclusion: $R_2 = 2R_1$; $\omega = 1/\tau = 1/RC$.

c. The transfer function of both RC-sections is found to be $H = (1 - j\omega RC)/(1 + j\omega RC)$. The oscillation condition is

$$\left(\frac{1 - j\omega RC}{1 + j\omega RC}\right)^2 \left(-\frac{R_1}{R_2}\right) = 1.$$

or: $R_1(1 + 2j\omega RC - \omega^2R^2C^2) = -R_2\,(1 - 2j\omega RC - \omega^2R^2C^2)$

From this it follows that: $R_1 = R_2$ (constant amplitude);

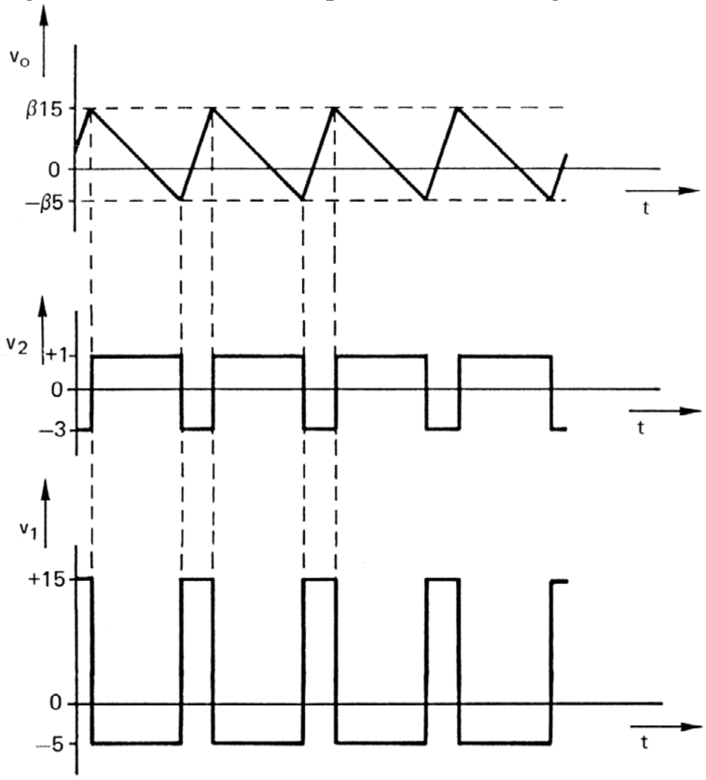
$\omega RC = 1$ (oscillation frequency $f = 1/2\pi RC$).

16.4. From the oscillation condition it follows that: $R_4 = 2R_3$ (see Eq. (16.6), so $R_{th}(T) = 1$ kΩ. $R_{th}(T)$ is the resistance value of the NTC when heated up to temperature T. If $T_0 = 273$ K, the thermistor has a resistance value of 1 kΩ at $T = 311$ K. The temperature rise which is 11 K, is caused by the dissipated power of $\Delta T/100 = 100$ mW $= v^2/R_{th}$ $\rightarrow \hat{v} = \sqrt{R_{th}\cdot 110\cdot 10^{-3}}/\sqrt{2}$ so the output amplitude is 14.8 V.

Voltage generators

16.5. The amplitude of v_2 remains the same, as does the slope of v_o. The voltage v_o increases (or decreases) up to (or down to) the switching levels of the Schmitt-trigger. At increasing R_1, the hysteresis increases too, so the frequency of the output decreases and the amplitude increases.

16.6. During the upgoing part of v_o, v_2 is negative hence v_1 is positive ($v_1 = V^+ = 15$ V); during the downgoing part v_1 is negative ($v_1 = V^- = -5$ V). The slope is $-v_2/R_5C = +(R_4/R_3)v_1/R_5C$. The duty cycle is found from the ratio between the charge and the discharge time intervals, so it is the ratio between the slope values. This is 3; the duty cycle of v_1 is 1:4 = 25%, that of v_2 (the inverse of v_1) is 75 %.



16.7. Amplitude: the positive peak is $\beta V^+ = 7.5$ V; the negative peak is $\beta V^- = -2.5$ V, which means that there is a peak value of 10 V.
Average value: the mean of the peak values, so $v_{o,m} = \frac{1}{2}(\beta V^+ + \beta V^-) = 2.5$ V.
Frequency: the upgoing slope is $3/R_5C = 3 \cdot 10^3$ V/s. The time period of this part is $10(\text{V})/3 \cdot 10^3$ (V/s) = 10/3 ms. The time interval of the downgoing part is three times as much, so 40/3 ms. The frequency is 3/40 kHz = 74 Hz.

16.8. V_{ref1} is negative; when the switch is on, v_o increases linearly in time up to a positive value that is determined by the upper switching level of the Schmitt-trigger. This level amounts to

$$V_{\text{ref}2} = \frac{R_2}{R_1 + R_2} + v_s \frac{R_1}{R_1 + R_2};$$

for the upper level, $v_s = V^+ = 18$ V, hence the peak value of the ramp voltage is $v_t = 7 \cdot (12/15.9) + 18 \cdot (3.9/15.9) \approx 9.7$ V. The lower switching level is 2.3 V. The slope is V_{ref1}/RC; after 1 ms, the peak value should be reached, hence $(V_{\text{ref1}}/RC) \cdot 10^{-3} = 9.7$, thus $R = 10.3$ kΩ.

16.9. When the switch goes on, v_o is equal to the voltage at the negative input terminal of the operational amplifier, which is zero (virtually grounded). For an average ramp voltage equal to zero, the non-inverting input of the operational amplifier should be connected to a voltage source the value of which is equal to the average value of the current situation, hence $-\frac{1}{2}v_t = -4.85$ V.

16.10. Due to the turn-on delay time of the switch, v_o continues rising during 2 ms after having reached the upper switching level of the Schmitt-trigger, with the same slope V_{ref1}/RC = 1000 V/s. The peak of 9.7 V is reached after 9.7 ms; the switch goes on after 11.7 ms, corresponding to a frequency of 85.5 Hz. The amplitude is $1000 \cdot 11.7 \cdot 10^{-3}$ = thus 11.7 V.

16.11. Take a positive reference voltage V_{ref1}, combined with a voltage source in series with the non-inverting input terminal of the operational amplifier that acts as an integrator; the latter should have a value that exceeds the negative switching level of the Schmitt-trigger. The switch control circuit should turn on the switch at a voltage $v_s = V^+ = 18$ V, and turn it off for lower values.

16.12. Except for the first period after switching on the circuit, v_c satisfies the equations (see section 8.1.1):

upgoing part: $v = (V^+ - \beta V^-)(1 - e^{-t/\tau}) + \beta V^-$ (1)

downgoing part: $v = (\beta V^+ - V^-)e^{-t/\tau} + V^-$ (2)

Here, the switching occurs at $t = 0$. Equation (1) is valid up to $v_c = \beta V^+$; if one assumes that this happens at $t = \tau_1$, then: $\beta V^+ = (V^+ - \beta V^-)(1 - e^{-\tau_1/\tau}) + \beta V^-$, from which follows:

$$-e^{-\tau_1/\tau} = \frac{\beta V^+ - V^+}{V^+ - \beta V^-}, \text{ or: } \tau_1 = \tau \ln \frac{V^+ - \beta V^-}{V^+ - \beta V^+}.$$

Similarly, in the case of the downgoing voltage:

$$\tau_2 = \tau \ln \frac{\beta V^+ - V^-}{\beta V^- - V^-}.$$

The duty cycle is $\tau_1/(\tau_1 + \tau_2)$, hence, for a duty cycle of 50%, $\tau_1 = \tau_2$. Thus $V^+ = V^-$, independent of β.

17. Modulation

Amplitude modulation and demodulation

17.1. See Section 17.1.2 and Figure 17.4; see also Figure 17.1(f)

17.2. The frequency spectrum of a symmetrical triangular wave contains the fundamental frequency and its odd harmonics, so $(2n + 1) \cdot 100$ Hz (n is an integer). Modulation produces sum- and difference frequencies of the carrier frequency (5 kHz) and the input signal, which is why the modulated signal is composed of components with frequencies 4900 and 5100 Hz, 4700 and 5300 Hz, 4500 and 5500 Hz and so on.

17.3. The signal bands may not overlap. The bandwidth of one modulated signal is the width over the two side bands, so $2 \cdot 500 = 1000$ Hz per channel. The system bandwidth should be $12 \cdot 1000 = 12$ kHz.

17.4. Assume $v_i = \hat{v}_i \cos(\omega t + \varphi)$. The bridge output voltage is $v_o = v_i(\Delta R/R)$, so $\hat{v}_o = \hat{v}_i(\Delta R/R)$. After amplification by a factor A and multiplication with the synchronous signal $v_s = \hat{v}_s \cos(\omega t + \varphi)$, the result is signal $A\hat{v}_o \cos(\omega t + \varphi)\hat{v}_s \cos(\omega t + \varphi) = A(\Delta R/R)\hat{v}_i\hat{v}_s[\frac{1}{2}(1 - \cos 2(\omega t + \varphi))]$

The DC component is $\frac{1}{2}A\hat{v}_i\hat{v}_s(\Delta R/R)$ = 2 V. For $\Delta R/R = -10^{-5}$, this is −2 V.

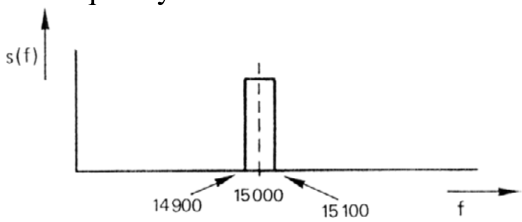
17.5. The amplified output signal Av_o is multiplied by $s(t)$. Only the term cos ωt produces a DC voltage (difference frequency zero). This part of the product equals

$$A\hat{v}_o \cos\omega t \cdot \frac{4}{\pi}\cos\omega t = A\frac{4}{\pi}\hat{v}_o\left[\frac{1}{2}(1-\cos 2\omega t)\right]$$

The DC component is $(2/\pi)A\hat{v}_i(\Delta R/R) = 2/\pi$ V, or $-2/\pi$ V.

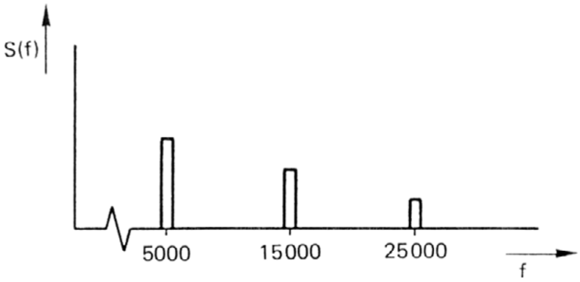
Systems based on synchronous detection

17.6. All components multiplied by the reference signal of 15 kHz and resulting in a frequency below 100 Hz contribute to the output signal. These are components within the frequency band of 14900 to 15100 Hz.



17.7. The quality factor of a bandpass filter is defined as the ratio between the resonance frequency and the bandwidth (Section 13.2.1). According to this definition, the quality factor of the synchronous detector amounts to 15000/200 = 75.

17.8. $s(t)$ contains the fundamental frequency $1/T$ = 5 kHz and odd harmonics. All components in v_i, multiplied by $s(t)$ and resulting in a frequency below 200 Hz contribute to the output signal. These are components within the frequency bands 4800–5200 Hz, 14800–15200 Hz, 24800–25200 Hz, and so on.

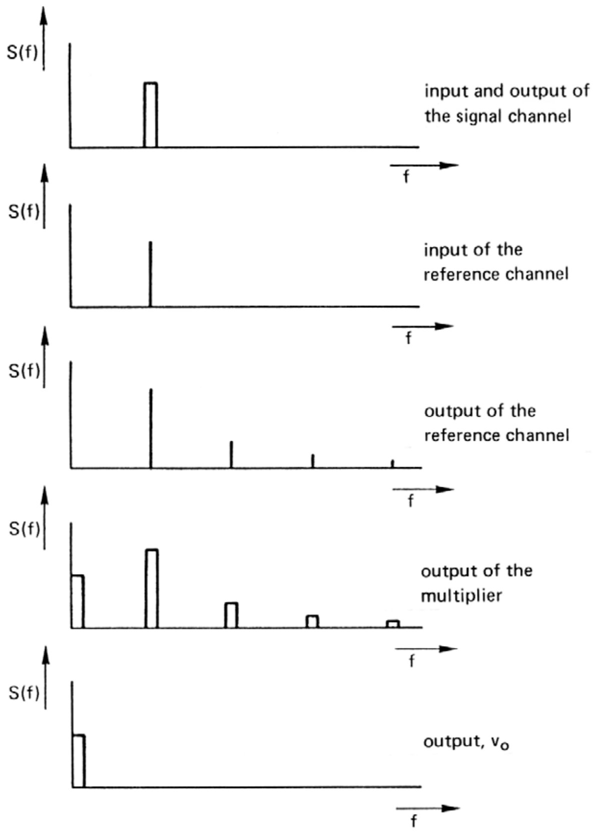


As the amplitude of the components in $s(t)$ is inversely proportional to the frequency, the height of the bands is also inversely proportional to the frequency.

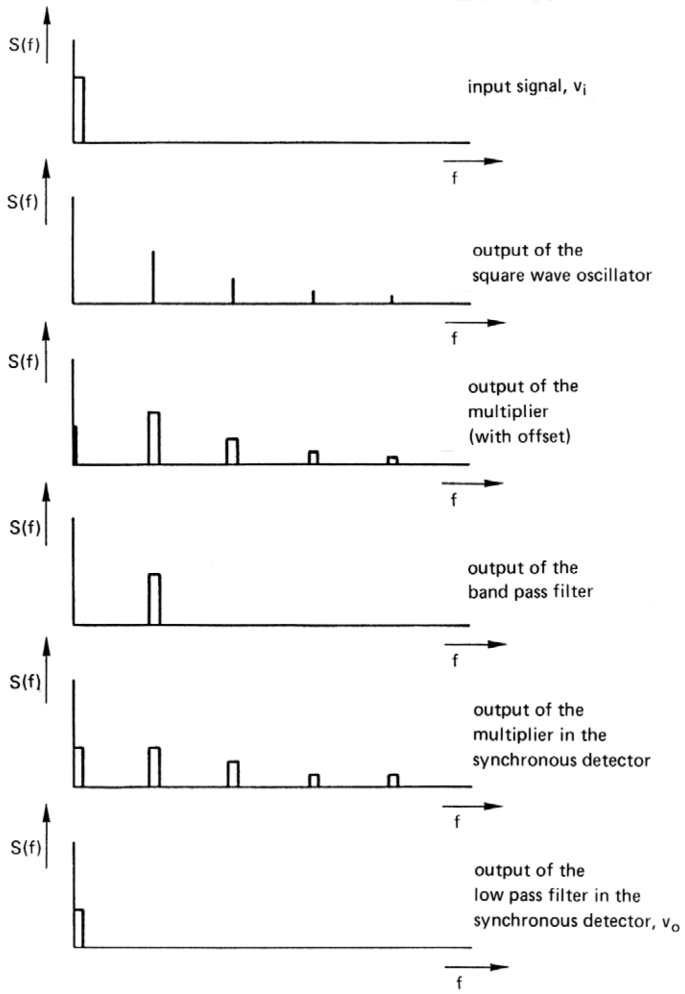
17.9. At constant VCO frequency, the input signal of H equals $v_i = \hat{v}_i \cos\omega t$, and the output signal is $|H|\hat{v}_i \cos(\omega t + \varphi)$, with $\varphi = \arg H$. Synchronous detection with v_i produces the y-signal $v_y = \frac{1}{2}|H|\hat{v}_i^2 \cos\varphi$; synchronous detection with a $\pi/2$ rad shifted input signal ($v_i \sin \omega t$) produces the x-signal $v_x = \frac{1}{2}|H|\hat{v}_i^2 \sin\varphi$. As $|H| \cos \varphi = \mathrm{Re}(H)$ and $|H| \sin \varphi = \mathrm{Re}(H)$, the screen shows the value of H at frequency ω in the complex plane. When the frequency of the VCO is controlled by a ramp generator, the spot on the screen describes the polar plot of H (see section 6.2). A periodic ramp results in a continuous, stable picture on the screen.

17.10. There are no requirements for the reference voltage, because the comparator produces a fixed output amplitude. If the comparator is omitted, the output of the synchronous detector depends on the amplitude of V_{ref}. This should have an accurate, stable value.

17.11.


S(f)
input and output of
the signal channel
f
S(f)
input of the
reference channel
f
S(f)
output of the
reference channel
f
S(f)
output of the
multiplier
f
S(f)
output, v_o
f

17.12.


S(f)
input signal, v_i
f
S(f)
output of the
square wave oscillator
f
S(f)
output of the
multiplier
(with offset)
f
S(f)
output of the
band pass filter
f
S(f)
output of the
multiplier in the
synchronous detector
f
S(f)
output of the
low pass filter in the
synchronous detector, v_o
f

18. Digital-to-analogue and analogue-to-digital conversion

Parallel converters

18.1

binary	1010111	101111111	100000001	10001111	100010001	1101111	1001001
octal	127	577	401	217	421	157	111
decimal	87	383	257	143	273	111	73
hexadecimal	57	17F	101	8F	111	6F	49

18.2. The MSB is equivalent to 5 V; each next bit is half the preceding bit, so $v_o = 5 + 2.5 + 1.25 + 0.625 + 0.3125 = 9.6875$ V.

18.3. 1 LSB = $1/2^{12} \approx 250$ ppm (= $250 \cdot 10^{-6}$). Over a temperature range of 80° the inaccuracy amounts to $80 \cdot 2 = 160$ ppm, corresponds to $160/250 = 0.64$ LSB.

18.4. See Section 18.1.2.

18.5. Electronic switches have well-defined and stable on and off states; other electronic parameters are much less stable.

18.6. The acquisition time for 1 bit is about $1/f = 5\ \mu$s; a 10-bit word requires 50 μs.

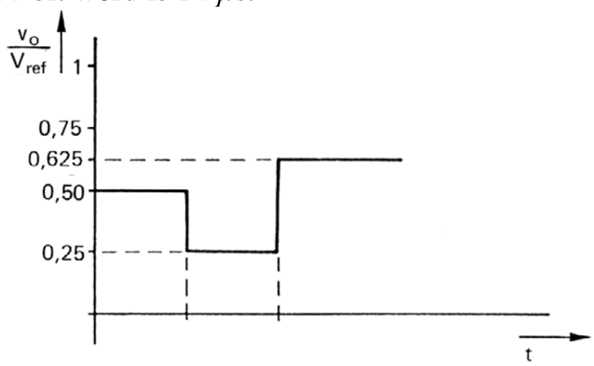
18.7. $v_o = GV_{ref}$; V_{ref} is the factor by which G (the input voltage of the DAC) is multiplied.

18.8. The two diodes connected in anti-parallel limit the input voltage of the operational amplifier to +0.6 and –0.6 V, and protect the input circuit against overvoltage.

Special converters

18.9. One bit requires 1 μs; ignoring other delay times in the circuit, the conversion time of a 14-bit word is 14 μs.

18.10.



18.11. The output code in the ideal case is 001 (MSB = 0; LSB = 1). The input voltages of the three comparators are $v_i = 0.630$ V, $v_i' = 1.260$ V and $v_i'' = 2.520$ V. These voltages are compared with $\frac{1}{2}V_{ref} = 2.500$ V.

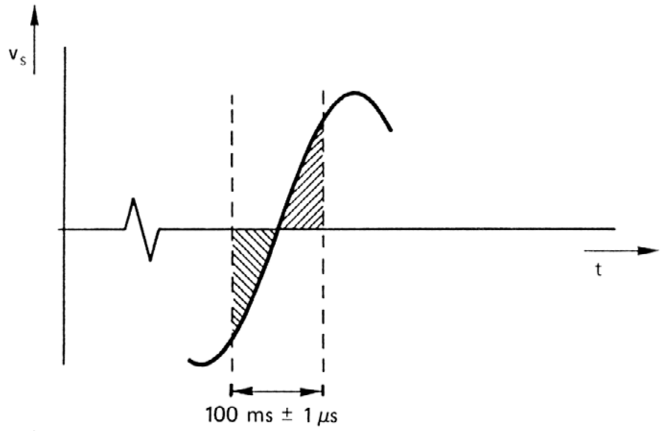
a. The comparators are accurate up to ±6 mV; in all cases the input voltage difference is more than 6 mV (minimum 20 mV), hence the code is correct.

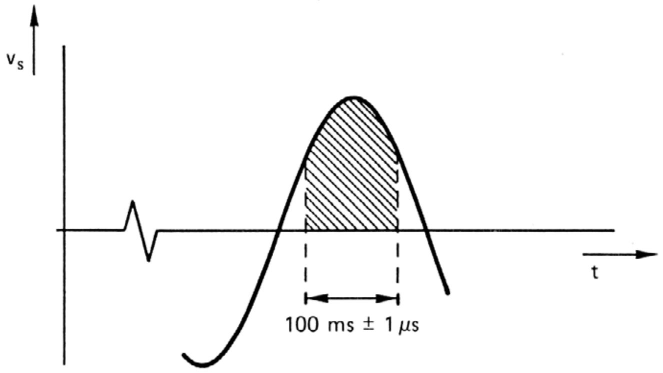
b. $v_i' = 2(v_i + V_{off})$; $v_i'' = 2(v_i' + V_{off}) = 4v_i + 6V_{off}$; v_i'' can be between (2520 + 36) mV and (2520 – 36) mV. In the last case the LSB can have the incorrect value of 0.

c. $v_i' = (2 \pm 0.01)v_i$; $v_i'' = (2 \pm 0.01)v_i' \approx (4 \pm 0.04)v_i$. v_i'' lies between (2520 + 25.2) mV and (2520 – 25.2) mV. In the latter case the LSB can have the incorrect value of 0. Apparently, the error depends on the value of v_i.

18.12. During the integration of both v_i and V_{ref} the frequency remains the same; the only requirement is a stable frequency during the conversion time.

18.13. The integration period is not exactly a multiple (here 5) of the interference signal period. There are two extreme cases:





The second situation shows the largest hatched area; the error equals one of these areas (10^{-6} Vs) so $\pm 10^{-5}$ (a constant input signal of 1 V that is integrated during 100 ms results in an indication of just 1 V).

19. Digital electronics

Digital components

19.1. a. $x + xy = x$ (absorption law)

b. $\bar{x} + xy = (\bar{x} + x)(\bar{x} + y)$ (distributive law)

$= 1 \cdot (\bar{x} + y) = \bar{x} + y$ (negation law and modulus law)

c. $\bar{x} + \overline{xy} = \bar{x} + (\bar{x} + \bar{y})$ (De Morgan's theorem)

$= \bar{x} + \bar{y}$ (law of equality)

d. $x \cdot (x \oplus y) = x \cdot (x\bar{y} + \bar{x}y)$ (can be proved with a truth table)

$= xx\bar{y} + x\bar{x}y$ (distributive law)

$= x\bar{y} + 0 \cdot y$ (law of equality and negation law)

$= x\bar{y}$ (modulus law)

e. $x \cdot (y + z) + x \cdot \bar{y} \cdot \bar{z} = x \cdot (y + z) + x \cdot \overline{(y + z)}$ (De Morgan's theorem)

$= x \cdot (y + z + \overline{y + z})$ (distributive law)

$= x \cdot 1 = x$ (negation law and modulus law).

19.2. a.

a	b	$a \oplus b$	$ab(a \oplus b)$
0	0	0	0
0	1	1	0
1	0	0	0
1	1	0	0

b.

a	b	c	$u = a + \bar{b} + \bar{c}$	$v = \bar{a} + \bar{b} + c$	$w = b + c$	uwv
0	0	0	1	1	0	0
0	0	1	1	1	1	1
0	1	0	1	1	1	1
0	1	1	0	1	1	0
1	0	0	1	1	0	0
1	0	1	1	1	1	1
1	1	0	1	0	1	0
1	1	1	1	1	1	1

19.3. $u = 1$ only if $a = 1$ and $b = 0$; $v = 0$ only if $\bar{v} = 1$, thus if $b = 1$ and $a = 0$; $w = 1$ only if $\bar{w} = 0$, thus if $a = b = c = 0$. Furthermore, $y = 0$ only if $v = w = 0$, and $x = 1$ only if $u = y = 1$.

a	b	c	u	v	w	y	x
0	0	0	0	1	1	1	0
0	0	1	0	1	0	1	0
0	1	0	0	0	0	0	0
0	1	1	0	0	0	0	0
1	0	0	1	1	0	1	1
1	0	1	1	1	0	1	1
1	1	0	0	1	0	1	0
1	1	1	0	1	0	1	0

19.4. From the truth table it appears that this flipflop only has a set and reset function. So the input combination $j = k = 1$ of the JK-flipflop must be made impossible. This can be achieved, for instance, by making k the inverse of j, using a NOT-gate: $D = J = \bar{K}$ (D is the input).

Logic circuits

19.5. For $e = 1$ at least one of the inputs of each AND gate is 0, so all outputs are 0; $y = 0$ irrespective of the other input values (indicated by – in the table).

The AND gates act as switches for d_n. The output is 0 for $d_n = 0$ and 1 if d_n and the other four inputs are 1. This is only true for $e = 1$ and just one combination of s_0, s_1 and s_2, which is different for each gate.

e	s_2	s_1	s_0	y
1	–	–	–	0
0	0	0	0	d_0
0	0	0	1	d_1
0	0	1	0	d_2
0	0	1	1	d_3
0	1	0	0	d_4
0	1	0	1	d_5
0	1	1	0	d_6
0	1	1	1	d_7

19.6. The number of gates connected in series increases with increasing numbers of bits, so that the total delay time increases, in particular for the MSB. Further, the signals are subject to different delay times, so the outputs make several transitions before reaching the steady state.

19.7. FF_0 is in the toggle mode ($J = K = 1$). FF_1 is in the toggle mode if $a_0 = 0$. The modes of FF_2 are listed in the table below.

a_0	a_1	J	K	q
0	0	0	0	(hold)
0	1	0	1	(reset)
1	0	0	0	(hold)
1	1	1	0	(set)

a_0	a_1	a_2	decimal value
1	1	0	3
0	0	1	4
1	0	1	5
0	1	1	6
1	1	0	3
0	0	1	4
1	0	1	5
.	.	.	.
.	.	.	.
.	.	.	.

The flipflops are of the master-slave type, so the values of J and K at the start of the clock pulse determine the output at the end of the clock pulse.

20. Measurement instruments

Electronic measurement instruments

20.1. The three meters are calibrated for the rms value of sine wave voltages. The correcting factor of meter B is $\pi\sqrt{2}/4 \approx 1.11$ (see also Section 2.1.2). Meter C indicates the peak value of a sine wave, when not corrected for rms values. The correction factor is thus $\frac{1}{2}\sqrt{2}$.

a. Sine wave: all meters indicate the correct value: $\frac{1}{2}\sqrt{2}\cdot 10 = 7.07$ V.

b. Square wave: the rms value, the mean of the modulus and the mean of the clamped voltage are all equal to the peak value; the indication of the meters is therefore: A: 10 V; B: 1.11·10 = 11.1 V: C: $\frac{1}{2}\sqrt{2}\cdot 10 = 7.07$ V.

c. Triangle: A triangular voltage whose peak value is $\hat{v}$ has an rms value of $(1.3)\sqrt{3}\cdot\hat{v}$; the mean of its modulus is $\frac{1}{2}\hat{v}$ and the mean of the clamped voltage is equal to the peak value. The indicated values are thus: A: $(1/3)\sqrt{3}\cdot 10 = 5.77$ V; B: 1.11·5 = 5.55 V and C: $\frac{1}{2}\sqrt{2}\cdot 10 = 7.07$ V.

20.2. In the two-wire mode, the resistances of the wires are included in the measured value $R + 2r$, with r the resistance of a single wire. For small values of R, the error would be unacceptably large. In the four-wire mode, a current is applied to the resistance through one pair of wires, whereas the voltage is measured via two other wires. The current through these latter wires is zero, so their resistance is not relevant; neither is the resistance of the first pair of wires, because the voltage is measured across the leads of the resistor.

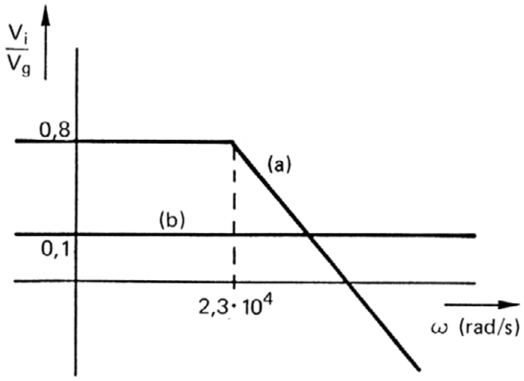
20.3. See Section 20.1.2. When a large time base period is chosen, the images appear clearly one after another.

20.4. The transfer of the circuit in Figure 20.6a is:

$$\frac{v_i}{v_g} = \frac{R_i}{R_i + R_g} \cdot \frac{1}{1 + j\omega R_p C_s},$$

where $R_p = R_i // R_g = 200$ kΩ and $C_s = C_i + C_k = 216$ pF.

At low frequencies, the transfer is 1/1.25 = 0.8. The break point of the characteristic is $\omega_k = 1/R_p C_s = 2.3 \cdot 10^4$ rad/s, corresponding with 3661 Hz.



The transfer of the circuit depicted in Figure 20.6b is, under the condition $R_g \ll R$:

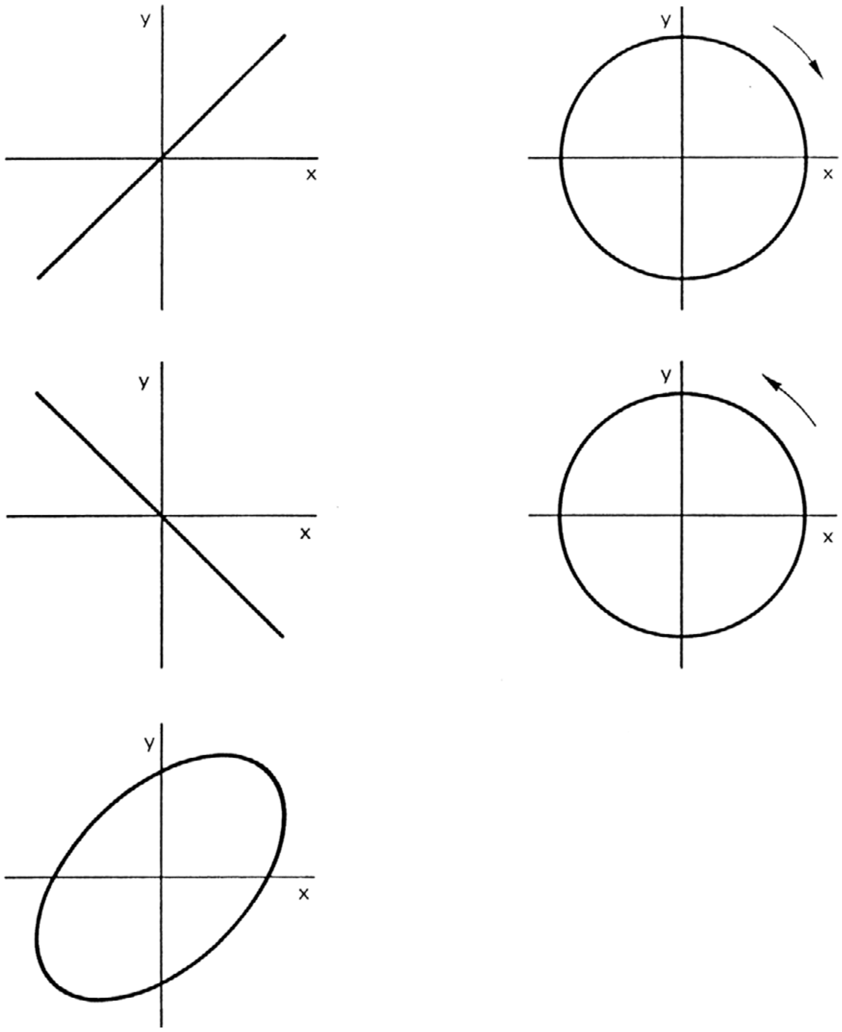
$$\frac{v_i}{v_g} = \frac{R_i}{R_i + R} \cdot \frac{1 + j\omega RC}{1 + j\omega R_p' C_s'},$$

with $R_p' = R_i // R = 0.9$ MΩ and $C_s' = C_k + C_i + C = 240$ pF.

Apparently, $RC = R_p' C_s'$, so the transfer does not depend on the frequency, and it equals $R_i/(R_i + R) = 0.1$.

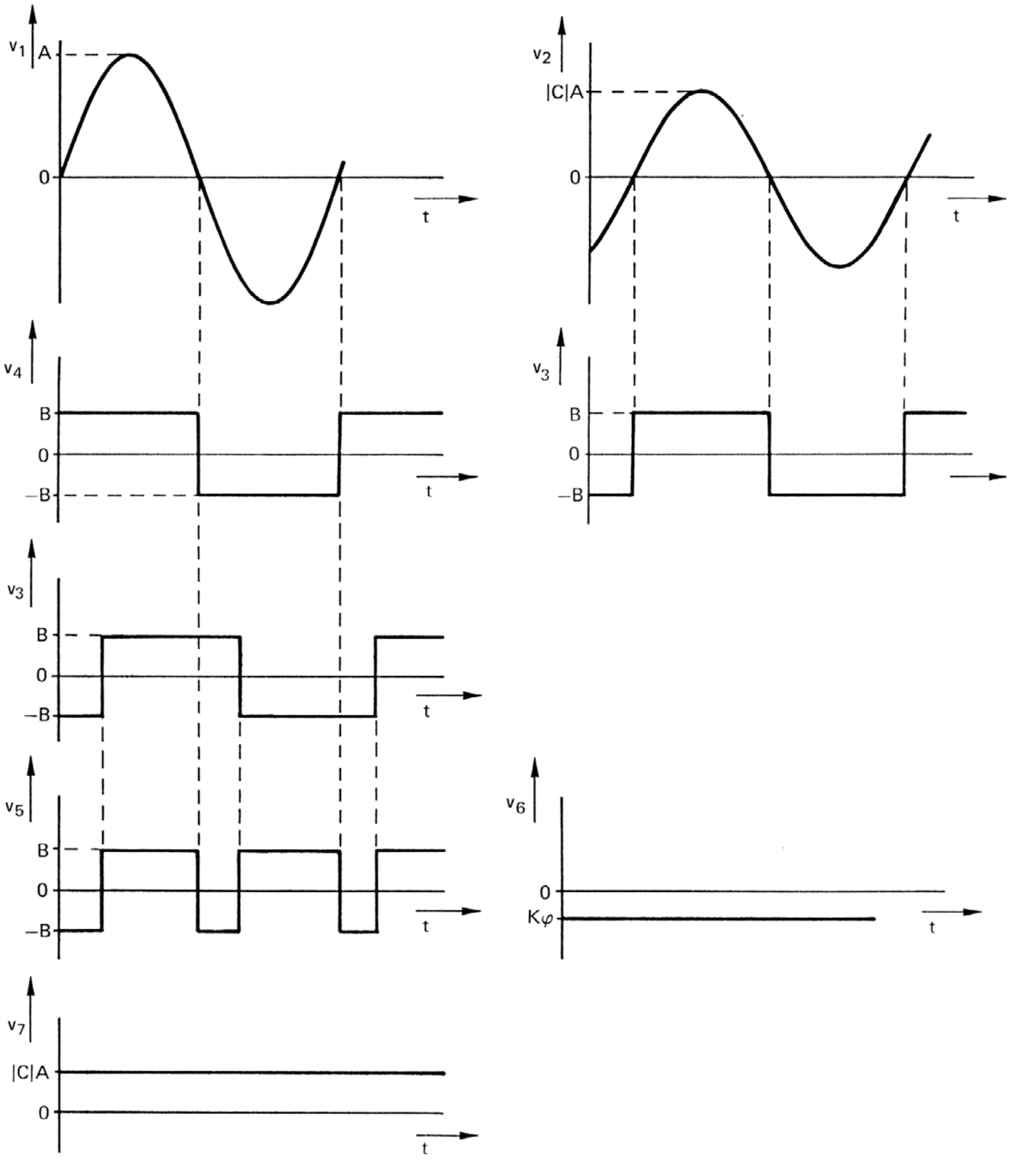
20.5. Without the attenuator the signal transfer is frequency dependent, corresponding with a low-pass characteristic. This results in the first of the three pictures (see also figure 8.3). The transfer is compensated by a network with a high-pass characteristic (with adjustable capacitance). Too high capacitance results in overcompensation, depicted in the middle of the figure. Only at correct compensation, the transfer is independent of the frequency; this will correspond to the rightmost picture.

20.6.



20.7. Logical circuits produce binary signals (0 or 1; high or low). A synchronous noise signal produces a random value (0 or 1) after each clock pulse. There are also noise generators with random transition time.

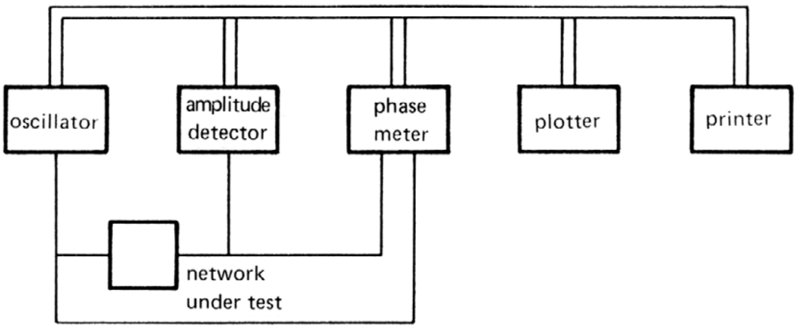
20.8.



Computer-based measurement instruments

20.9.

a.



b. Four decades, spread over 40 cm with a resolution of 0.4 mm means 1000 steps for a factor of 10^4, or $\sqrt[1000]{10^4} \approx 1.01$ for each step.

c. 20 cm with a resolution of 0.4 mm means 500 steps; for 40 dB: 0.08 dB per step.

d. 500 steps to cover 2π rad: 2π/500 = 0.0126 rad = 0.72° per step.

20.10. See Section 20.2.1.2.

20.11. See Figure 20.14.

20.10. See Section 20.2.3.

21. Measurement errors

Types of measurement errors

21.1. a. The absolute error is 0.01·(788 + 742) = 15.3 mV.

The relative error is $\frac{15.3}{788-742}=\frac{15.3}{46}=33.3\%$

b. The differential voltage is 46 mV; the relative error is 1%; the absolute error is 1% of 46 mV, which is 0.46 mV.

21.2. a. The first measurement value shows a deviation of a factor of 10, probably due to a reading mistake. This value will be disregarded. As the measurement is performed repeatedly with the same instrument, systematic errors cannot be recognized. Obviously there are random errors.

b. The best estimation is the mean (of the four remaining values): I_m = 24.86/4 = 6.215 mA. A better notation is 6.2 mA, according to the fluctuations.

c. The random error can be lowered by a larger number of measurements. Systematic errors cannot be reduced by this method.

d. The best estimation of the current value is 6.25 mA, the middle of the tolerance band. The difference between this and the best estimation from the first series of measurements is 0.035 mA. Apparently, the first measurement is subject to a systematic error of about 0.04 mA.

21.3. $v_o=-\frac{R_2}{R_1}\cdot v_\mathrm{i}=-\frac{100}{3.9}\cdot 0.2=-5.128\,\mathrm{V}.$

The additive error due to V_off and I_bias is:

$$\Delta v_\mathrm{o}=\left(1+\frac{R_2}{R_1}\right)V_\mathrm{off}+I_\mathrm{bias}R_2=40\,\mathrm{mV}+10\ \mathrm{mV}\ =\ 50\ \mathrm{mV}.$$

The multiplicative error due to tolerances of v_i, R_1 and R_2 amounts to:

$$\frac{\Delta v_\mathrm{o}}{v_\mathrm{o}}=\frac{\Delta R_1}{R_1}+\frac{\Delta R_2}{R_2}+\frac{\Delta v_\mathrm{i}}{v_\mathrm{i}}=0.5+0.5+0.2=1.2\%.$$

The maximum error is $50 + 0.012\cdot 5.13\cdot 10^3$ = 112 mV. The output voltage has a value somewhere between –5.242 and –5.018 V. The correct indication of the output voltage is –5.13 ± 0.11 V.

21.4. a. The measured (nominal) resistance is $R = V_\mathrm{o}/I_1$ = 75 Ω.

The multiplicative error is: $\Delta R/R = \Delta v_\mathrm{o}/v_\mathrm{o} + \Delta I/I$ = 0.5 + 0.5 = ± 1% ≡ 0.75 Ω, with $\Delta v_\mathrm{o}/v_\mathrm{o}$ the relative error in v_o.

The total resistance of the two connection wires is between 0 and 6 Ω, denoted as 3 ± 3 Ω.

The additive error is $\pm(\Delta V_\mathrm{o}/I_1) + 2(r \pm \Delta r)$ = ±0.5 + (3 ± 3) = 3 ± 3.5 Ω, where ΔV_o is the absolute error in V_o. In total: R = 75 – 3 ± 4.25 = 72 ± 4.25 Ω.

b. $V_\mathrm{o} = I_1(R + r_1) - I_2r_3$; assuming that $I_1r_1 = I_2r_3$, V_o equals I_1R, so the (nominal) resistance is $R = V_\mathrm{o}/I_1$ = 70 Ω.

The multiplicative error is (as in a.): ±1% ≡ ±0.7 Ω.

The additive error is:

$$\frac{\Delta V_o}{I_1} + \frac{\Delta(I_1 r_1 - I_2 r_3)}{I_1} = \frac{\Delta V_o}{I_1} + \frac{2(I\Delta r + r\Delta I)}{I}$$

$$\approx \frac{\Delta V_o}{I_1} \pm 2\Delta r = \pm 0.5 \pm 2 = \pm 2.5\Omega$$

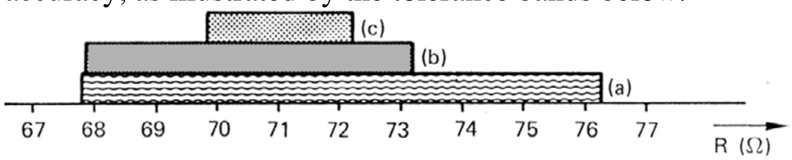
Thus $R = 70\ \Omega \pm 3.2\ \Omega$.

c. Nominal value: $V_o = I_1 R$, thus $R = V_o/I_1 = 71\ \Omega$.

Multiplicative error (as in a.): ±1% ≡ ± 0.71 Ω.

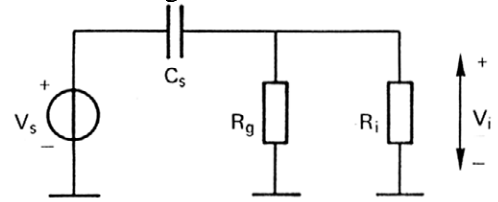
Additive error: $\pm\Delta V_o/I_1 \equiv \pm 0.5\ \Omega$, so $R = 71\ \Omega \pm 1.2\ \Omega$.

The three measurement methods for the same resistance value have ascending accuracy, as illustrated by the tolerance bands below.



Measurement interference

21.5. The transfer from the error source to the input of the measurement system can be deduced using the circuit model shown below.



Applying the rules for the complex variables we find:

$$V_i = \frac{R_p}{1/j\omega C_s + R_p} V_s = \frac{j\omega R_p C_s}{1 + j\omega R_p C_s} V_s, \text{ with } R_p = R_g // R_i$$

In all cases, $\omega R_p C_s << 1$ ($\omega = 2\pi f \approx 314$; $R_p < 10^7$ and $C_s < 10^{-13}$) so: $V_i = j\omega R_p C_s V_s$. The rms value of the input voltage due to V_s becomes: $V_i = 2\pi f R_p C_s V_s$. As the peak values are identical to the amplitudes (mean value is zero) the peak value of the input voltage is $\hat{v}_i = 2\pi f R_p C_s \hat{v}_s$

The input voltage due to the signal source V_g is

$$\frac{R_i}{R_i + R_g} V_g = 0.99 V_g.$$

The signal-to-noise ratio is the ratio between the rms values of the input voltages due to V_g and V_s, respectively.

a. In the case of a disconnected signal source, $R_p = R_i$. So the peak value of V_i is: $\hat{v}_i = 2\pi \cdot 50 \cdot 10^7 \cdot 10^{-13} \cdot \sqrt{2} \cdot 220 \text{ mV} \approx 97.7 \text{ mV}$.

b. For a connected signal source, $R_p = R_g // R_i = 99$ kΩ. The rms value and the peak value of V_i (due to V_s) are found to be: $2\pi \cdot 50 \cdot 99 \cdot 10^3 \cdot 10^{-13} \cdot 220 = 0.68$ mV and 0.97 mV, respectively.

The signal-to-noise ratio becomes: $\dfrac{0.99 \cdot 100 \cdot 10^{-3}}{0.68 \cdot 10^{-3}} = 146.$

c. The capacitance is reduced to $0.01C_s = 10^{-15}$ F, resulting in an input voltage with rms value of $2\pi \cdot 50 \cdot 10^7 \cdot 10^{-15} \cdot 220 = 0.69$ mV and a peak value of 0.98 mV.

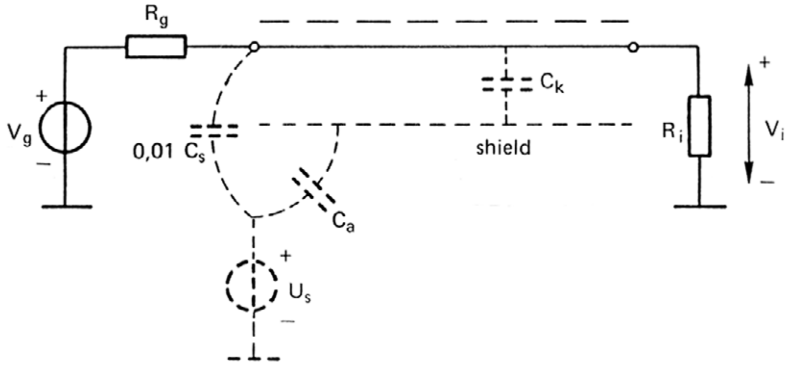
d. Similar to b., the rms value and the peak value of the input voltage (due to V_s) are found to be:

$2\pi \cdot 50 \cdot 99 \cdot 10^3 \cdot 10^{-15} \cdot 220 = 6.84$ μV and 9.68 μV, respectively. The corresponding signal-to-noise ratio is

$$\frac{0.99 \cdot 100 \cdot 10^{-3}}{6.84 \cdot 10^{-6}} = 14\,469.$$

e. The next figure shows a model of the situation: a capacitance between the error voltage source and the signal conductor (here $0.01C_s$); a capacitance between the error voltage source and the shield (C_a) and a capacitance between the shield and the signal conductor (the cable capacitance C_k).

Usually, the dimensions of the shield are much greater than those of the signal conductor, so $C_a \gg C_s$. A typical value for C_k is 100 pF per meter of cable length.



When the shield is grounded, C_a has no effect: error injection only occurs via $0.01C_s$. C_k is parallel to R_i, resulting in a low-pass signal transfer V_i/V_g (see also Figure 20.6), with a break point at 16.1 kHz (for C_k = 100 pF). If this is not acceptable, a buffer amplifier can be connected near to the signal source, or active guarding can be applied.

When the shield is not grounded, the total capacitance between the error voltage source and the signal conductor is $0.01C_s$//(C_a in series with C_k) or:

$$0.01C_s + \frac{C_a C_k}{C_a + C_k}$$

Usually, C_a is much larger than C_s, so the capacitance between the error voltage source and the signal conductor is much larger compared to a non-shielded conductor. Hence, the error voltage of a floating shield is larger than when there is no shield at all.

21.6. a. The mains filter behaves as a low-pass filter for signals between the conductors and the ground as well as for signals between the conductors. All frequency components above 50 Hz should be filtered out.

b. Because of the symmetry of the configuration, the voltage between the instrument case and the ground is just half the voltage of that between the two mains connections, hence 110 V.